

# CHAPTER 3

## Applications of Differentiation

### Section 3.1 Extrema on an Interval

Solutions to Odd-Numbered Exercises

1.  $f(x) = \frac{x^2}{x^2 + 4}$

$$f'(x) = \frac{(x^2 + 4)(2x) - (x^2)(2x)}{(x^2 + 4)^2} = \frac{8x}{(x^2 + 4)^2}$$

$$f'(0) = 0$$

5.  $f(x) = (x + 2)^{2/3}$

$$f'(x) = \frac{2}{3}(x + 2)^{-1/3}$$

$f'(-2)$  is undefined.

9. Critical numbers:  $x = 1, 2, 3$

$x = 1, 3$ : absolute maximum

$x = 2$ : absolute minimum

13.  $g(t) = t\sqrt{4-t}$ ,  $t < 3$

$$\begin{aligned} g'(t) &= t\left[\frac{1}{2}(4-t)^{-1/2}(-1)\right] + (4-t)^{1/2} \\ &= \frac{1}{2}(4-t)^{-1/2}[-t + 2(4-t)] \\ &= \frac{8-3t}{2\sqrt{4-t}} \end{aligned}$$

Critical number is  $t = \frac{8}{3}$ .

17.  $f(x) = 2(3-x)$ ,  $[-1, 2]$

$f'(x) = -2 \Rightarrow$  No critical numbers

Left endpoint:  $(-1, 8)$  Maximum

Right endpoint:  $(2, 2)$  Minimum

3.  $f(x) = x + \frac{27}{2x^2} = x + \frac{27}{2}x^{-2}$

$$f'(x) = 1 - \frac{27}{x^3} = 1 - \frac{27}{x^3}$$

$$f'(3) = 1 - \frac{27}{3^3} = 1 - 1 = 0$$

7. Critical numbers:  $x = 2$

$x = 2$ : absolute maximum

11.  $f(x) = x^2(x-3) = x^3 - 3x^2$

$$f'(x) = 3x^2 - 6x = 3x(x-2)$$

Critical numbers:  $x = 0, x = 2$

15.  $h(x) = \sin^2 x + \cos x$ ,  $0 < x < 2\pi$

$$h'(x) = 2 \sin x \cos x - \sin x = \sin x(2 \cos x - 1)$$

On  $(0, 2\pi)$ , critical numbers:  $x = \frac{\pi}{3}, x = \pi, x = \frac{5\pi}{3}$

19.  $f(x) = -x^2 + 3x$ ,  $[0, 3]$

$$f'(x) = -2x + 3$$

Left endpoint:  $(0, 0)$  Minimum

Critical number:  $(\frac{3}{2}, \frac{9}{4})$  Maximum

Right endpoint:  $(3, 0)$  Minimum

21.  $f(x) = x^3 - \frac{3}{2}x^2$ ,  $[-1, 2]$

$$f'(x) = 3x^2 - 3x = 3x(x - 1)$$

Left endpoint:  $\left(-1, -\frac{5}{2}\right)$  Minimum

Right endpoint:  $(2, 2)$  Maximum

Critical number:  $(0, 0)$

Critical number:  $\left(1, -\frac{1}{2}\right)$

25.  $g(t) = \frac{t^2}{t^2 + 3}$ ,  $[-1, 1]$

$$g'(t) = \frac{6t}{(t^2 + 3)^2}$$

Left endpoint:  $\left(-1, \frac{1}{4}\right)$  Maximum

Critical number:  $(0, 0)$  Minimum

Right endpoint:  $\left(1, \frac{1}{4}\right)$  Maximum

29.  $f(x) = \cos \pi x$ ,  $\left[0, \frac{1}{6}\right]$

$$f'(x) = -\pi \sin \pi x$$

Left endpoint:  $(0, 1)$  Maximum

Right endpoint:  $\left(\frac{1}{6}, \frac{\sqrt{3}}{2}\right)$  Minimum

23.  $f(x) = 3x^{2/3} - 2x$ ,  $[-1, 1]$

$$f'(x) = 2x^{-1/3} - 2 = \frac{2(1 - \sqrt[3]{x})}{\sqrt[3]{x}}$$

Left endpoint:  $(-1, 5)$  Maximum

Critical number:  $(0, 0)$  Minimum

Right endpoint:  $(1, 1)$

27.  $h(s) = \frac{1}{s-2}$ ,  $[0, 1]$

$$h'(s) = \frac{-1}{(s-2)^2}$$

Left endpoint:  $\left(0, -\frac{1}{2}\right)$  Maximum

Right endpoint:  $(1, -1)$  Minimum

31.  $y = \frac{4}{x} + \tan \frac{\pi x}{8}$ ,  $[1, 2]$

$$y' = \frac{-4}{x^2} + \frac{\pi}{8} \sec^2 \frac{\pi x}{8} = 0$$

$$\frac{\pi}{8} \sec^2 \frac{\pi x}{8} = \frac{4}{x^2}$$

On the interval  $[1, 2]$ , this equation has no solutions.  
Thus, there are no critical numbers.

Left endpoint:  $(1, \sqrt{2} + 3) \approx (1, 4.4142)$  Maximum

Right endpoint:  $(2, 3)$  Minimum

33. (a) Minimum:  $(0, -3)$

Maximum:  $(2, 1)$

(b) Minimum:  $(0, -3)$

(c) Maximum:  $(2, 1)$

(d) No extrema

35.  $f(x) = x^2 - 2x$

(a) Minimum:  $(1, -1)$

Maximum:  $(-1, 3)$

(b) Maximum:  $(3, 3)$

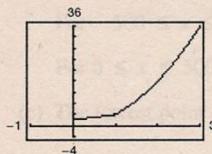
(c) Minimum:  $(1, -1)$

(d) Minimum:  $(1, -1)$

37.  $f(x) = \begin{cases} 2x + 2, & 0 \leq x \leq 1 \\ 4x^2, & 1 < x \leq 3 \end{cases}$

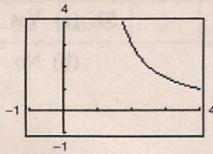
Left endpoint: (0, 2) Minimum

Right endpoint: (3, 36) Maximum

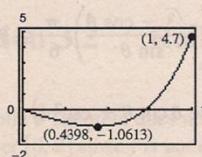


39.  $f(x) = \frac{3}{x-1}, (1, 4]$

Right endpoint: (4, 1) Minimum



41. (a)



Maximum: (1, 4.7) (endpoint)

Minimum: (0.4398, -1.0613)

(b)

$$f(x) = 3.2x^5 + 5x^3 - 3.5x, [0, 1]$$

$$f'(x) = 16x^4 + 15x^2 - 3.5$$

$$16x^4 + 15x^2 - 3.5 = 0$$

$$x^2 = \frac{-15 \pm \sqrt{(15)^2 - 4(16)(-3.5)}}{2(16)}$$

$$= \frac{-15 \pm \sqrt{449}}{32}$$

$$x = \sqrt{\frac{-15 + \sqrt{449}}{32}} \approx 0.4398$$

$$f(0) = 0$$

$f(1) = 4.7$  Maximum (endpoint)

$$f\left(\sqrt{\frac{-15 + \sqrt{449}}{32}}\right) \approx -1.0613$$

Minimum: (0.4398, -1.0613)

43.  $f(x) = (1 + x^3)^{1/2}, [0, 2]$

$$f'(x) = \frac{3}{2}x^2(1 + x^3)^{-1/2}$$

$$f''(x) = \frac{3}{4}(x^4 + 4x^3)(1 + x^3)^{-3/2}$$

$$f'''(x) = -\frac{3}{8}(x^6 + 20x^3 - 8)(1 + x^3)^{-5/2}$$

Setting  $f'''(x) = 0$ , we have  $x^6 + 20x^3 - 8 = 0$ .

$$x^3 = \frac{-20 \pm \sqrt{400 - 4(1)(-8)}}{2}$$

$$x = \sqrt[3]{-10 \pm \sqrt{108}} = \sqrt{3} - 1$$

In the interval  $[0, 2]$ , choose

$$x = \sqrt[3]{-10 + \sqrt{108}} = \sqrt{3} - 1 \approx 0.732.$$

$$\left|f''\left(\sqrt[3]{-10 + \sqrt{108}}\right)\right| \approx 1.47 \text{ is the maximum value.}$$

45.  $f(x) = (x + 1)^{2/3}, [0, 2]$

$$f'(x) = \frac{2}{3}(x + 1)^{-1/3}$$

$$f''(x) = -\frac{2}{9}(x + 1)^{-4/3}$$

$$f'''(x) = \frac{8}{27}(x + 1)^{-7/3}$$

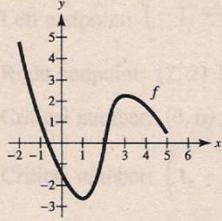
$$f^{(4)}(x) = -\frac{56}{81}(x + 1)^{-10/3}$$

$$f^{(5)}(x) = \frac{560}{243}(x + 1)^{-13/3}$$

$$|f^{(4)}(0)| = \frac{56}{81} \text{ is the maximum value.}$$

47.  $f(x) = \tan x$  $f$  is continuous on  $[0, \pi/4]$  but not on  $[0, \pi]$ .  $\lim_{x \rightarrow \pi/2^-} \tan x = \infty$ .

49.



51. (a) Yes

(b) No

53. (a) No

(b) Yes

55.  $P = VI - RI^2 = 12I - 0.5I^2, 0 \leq I \leq 15$

 $P = 0$  when  $I = 0$ . $P = 67.5$  when  $I = 15$ .

$P' = 12 - I = 0$

Critical number:  $I = 12$  ampsWhen  $I = 12$  amps,  $P = 72$ , the maximum output.No, a 20-amp fuse would not increase the power output.  
 $P$  is decreasing for  $I > 12$ .

57.

$S = 6hs + \frac{3s^2}{2} \left( \frac{\sqrt{3} - \cos \theta}{\sin \theta} \right), \frac{\pi}{6} \leq \theta \leq \frac{\pi}{2}$

$\frac{dS}{d\theta} = \frac{3s^2}{2} \left( -\sqrt{3}\csc \theta \cot \theta + \csc^2 \theta \right)$

$= \frac{3s^2}{2} \csc \theta (-\sqrt{3}\cot \theta + \csc \theta) = 0$

$\csc \theta = \sqrt{3}\cot \theta$

$\sec \theta = \sqrt{3}$

$\theta = \text{arcsec } \sqrt{3} \approx 0.9553 \text{ radians}$

$S\left(\frac{\pi}{6}\right) = 6hs + \frac{3s^2}{2}(\sqrt{3})$

$S\left(\frac{\pi}{2}\right) = 6hs + \frac{3s^2}{2}(\sqrt{3})$

$S(\text{arcsec } \sqrt{3}) = 6hs + \frac{3s^2}{2}(\sqrt{2})$

 $S$  is minimum when  $\theta = \text{arcsec } \sqrt{3} \approx 0.9553$  radians.

59. (a)  $y = ax^2 + bx + c$

$y' = 2ax + b$

The coordinates of  $B$  are  $(500, 30)$ , and those of  $A$  are  $(-500, 45)$ .  
From the slopes at  $A$  and  $B$ ,

$-1000a + b = -0.09$

$1000a + b = 0.06$ .

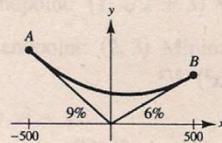
Solving these two equations, you obtain  $a = 3/40000$  and  $b = -3/200$ . From the points  $(500, 30)$  and  $(-500, 45)$ , you obtain

$30 = \frac{3}{40000} 500^2 + 500 \left( \frac{-3}{200} \right) + c$

$45 = \frac{3}{40000} 500^2 - 500 \left( \frac{-3}{200} \right) + c$

In both cases,  $c = 18.75 = \frac{75}{4}$ . Thus,

$y = \frac{3}{40000}x^2 - \frac{3}{200}x + \frac{75}{4}$ .



—CONTINUED—

**59. —CONTINUED—**

(b)

$x$	-500	-400	-300	-200	-100	0	100	200	300	400	500
$d$	0	.75	3	6.75	12	18.75	12	6.75	3	.75	0

For  $-500 \leq x \leq 0$ ,  $d = (ax^2 + bx + c) - (-0.09x)$ .

For  $0 \leq x \leq 500$ ,  $d = (ax^2 + bx + c) - (0.06x)$ .

(c) The lowest point on the highway is  $(100, 18)$ , which is not directly over the point where the two hillsides come together.

**61.** True. See Exercise 25.

**63.** True.

**End**