

# 1 Multiple Representation of Functions: Graphical, Numerical, and Analytical

## Key to Text Coverage

Section	Examples	Exercises	Topics
P.1	1–6	1–85	Graphs and models
P.3	1–5	1–76	Functions and their graphs
P.4	1–3	1–18	Fitting models to data

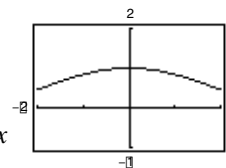
## Summary

There are three ways to present calculus concepts: graphically, numerically, and analytically. It is important to reinforce each of these approaches with the other two. For instance, the behavior of the function

$$f(x) = \frac{\sin x}{x}$$

near  $x = 0$  is of crucial importance in the development of limits and derivatives. You can analyze this behavior graphically by noting that the function seems to approach  $y = 1$  as  $x$  gets closer and closer to 0.

Or you can analyze this behavior numerically by constructing a table of values for  $x$  near 0.



$x$	-0.1	-0.01	-0.001	0	0.001	0.01	0.1
$\frac{\sin x}{x}$	0.9983	0.99998	0.9999998	?	0.9999998	0.99998	0.9983

Again, the function seems to be approaching 1 as  $x$  approaches 0. Finally, you can use *analytical* methods to study the function

$$f(x) = \frac{\sin x}{x}$$

as indicated on page 63 of the text. There a mathematical proof that  $f(x)$  approaches 1 as  $x$  tends to 0 is presented.

In other words, functions can be represented graphically, numerically, and analytically. Example 3 on page 32 illustrates how these three approaches can be used together to model a real life problem. In that example a model is given for the number of daylight hours at various dates of the year. There is a table of data values from a weather almanac, a graph illustrating the periodic nature of the data, and a trigonometric model for the data

$$f(t) = 728 - 73 \sin\left(\frac{2\pi t}{365} + \frac{\pi}{2}\right).$$

All three approaches are integrated in order to fully understand the model, and to use it for real life applications.

**Worked Example**

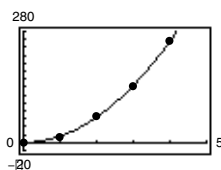
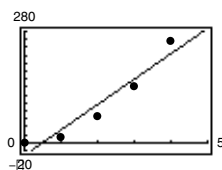
An observer obtains the following data giving the distance  $s$  in feet that an object has fallen  $t$  seconds after being released from a tall building.

$x$	0	1	2	3	4
$s$	0	15.5	64.4	143.2	256.4

- A linear model for the data is  $f(t) = 64.05t - 32.2$ . Graph this model and the data together and discuss how well the model fits the data.
- A quadratic model for the data is  $g(t) = 16.093t^2 - 0.321t - 0.014$ . Graph this model and the data together and compare the accuracy with the linear model above.
- Find the error in the quadratic model when  $t = 2$ .
- Use the quadratic model to estimate how far the object has fallen after 3.5 seconds.
- Use the quadratic model to estimate how long it takes for the object to fall 100 feet.

**SOLUTION**

- (a) The model is a poor fit.



- The quadratic model is much more accurate.
- $g(2) = 63.716$  and the error is  $64.4 - 63.716 = 0.684$ .
- $g(3.5) \approx 196$  feet.
- By graphing  $y = 100$  and  $g(t)$  in the same viewing window, you see that they intersect at  $t = 2.503$  seconds.

**Notes**

- This model is called the **least squares regression line** for the data and can be calculated with a graphing utility.
- You can also solve this problem by using the quadratic formula to solve the equation

$$16.093t^2 - 0.321t - 0.014 = 100.$$

Which method is easier?

Name \_\_\_\_\_

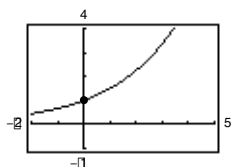
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**Sample Questions**

Show all your work on a separate sheet of paper. Indicate clearly the methods you use because you will be graded on the correctness of your methods as well as on the accuracy of your answers.

**Multiple Choice**

1. Determine the model that best fits the graph.



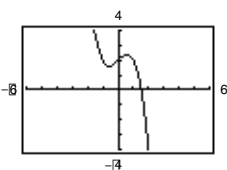
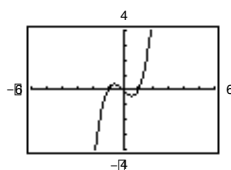
- (a)  $y = 1.5^x$       (b)  $y = x^2 + 1$       (c)  $y = x^4 + 1$       (d)  $y = \sec x$       (e)  $y = x + 1$

2. Determine the model that best represents the given data.

$x$	0	0.3	0.8	1.6	2.2	2.5
$y$	0	0.3	0.7	1.0	0.8	0.6

- (a)  $y = x$       (b)  $y = x^2$       (c)  $y = \cos x$       (d)  $y = \sin x$       (e)  $y = e^x$

3. The graph of
- $f$
- is shown on the left. What is the graph on the right?



- (a)  $-f(x)$       (b)  $f(x) + 2$       (c)  $-f(x + 2)$       (d)  $-f(x - 2)$       (e)  $-f(x) + 2$

**Free Response**

Consider the following data from an experiment.

$x$	-3	-2	0	1	4
$y$	8.5	7	4	2.5	-2

- (a) Express  $y$  as a linear function of  $x$ .  
 (b) Express  $x$  as a linear function of  $y$ .  
 (c) Do you think a quadratic model would represent this data better than a linear model? Why or why not?  
 (d) Graph  $y$  as a function of  $x$  and indicate the intercepts.  
 (e) Find the value of  $x$  if  $y = 5.4$ .

## SOLUTIONS

## Multiple Choice

1. Answer (a). Notice that the graph passes through (0, 1) and (1, 1.5).
2. Answer (d). The data seems periodic and passes through the origin (0, 0).
3. Answer (e). The graph is a reflection in the  $x$ -axis followed by an upward shift of two units.

## Free Response

- (a) One way to solve this problem is to use the point-slope formula for the line containing the third and fourth points.

$$y - 4 = \frac{2.5 - 4}{1 - 0}(x - 0)$$

$$y - 4 = -1.5x$$

$$y = -1.5x + 4$$

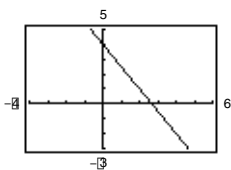
Notice that the other three points also lie on this line.

- (b) Solving  $y = -\frac{3}{2}x + 4$  for  $x$ , you obtain

$$\frac{3x}{2} = -y + 4 \text{ or } x = -\frac{2y}{3} + \frac{8}{3}.$$

- (c) No, the data is linear and a quadratic model would just have 0 as the coefficient of the  $x^2$ -term.

- (d) The intercepts are (0, 4) and  $(\frac{8}{3}, 0)$ .



- (e) If  $y = 5.4$ , then part (b) gives

$$x = -\frac{2y}{3} + \frac{8}{3} = -\frac{2(5.4)}{3} + \frac{8}{3} = -\frac{14}{15} \approx -0.9333.$$