LAB 9

Constructing an Arch Dam

Surface Area, Volume, and Fluid Force

When designing structures like dams, many questions arise. How much material will be needed to build the structure? If the material used to build the structure needs curing, like concrete does, what is the surface area? What forces act on the structure and how can the forces be calculated? Calculus is a helpful tool in answering these questions.

Observations

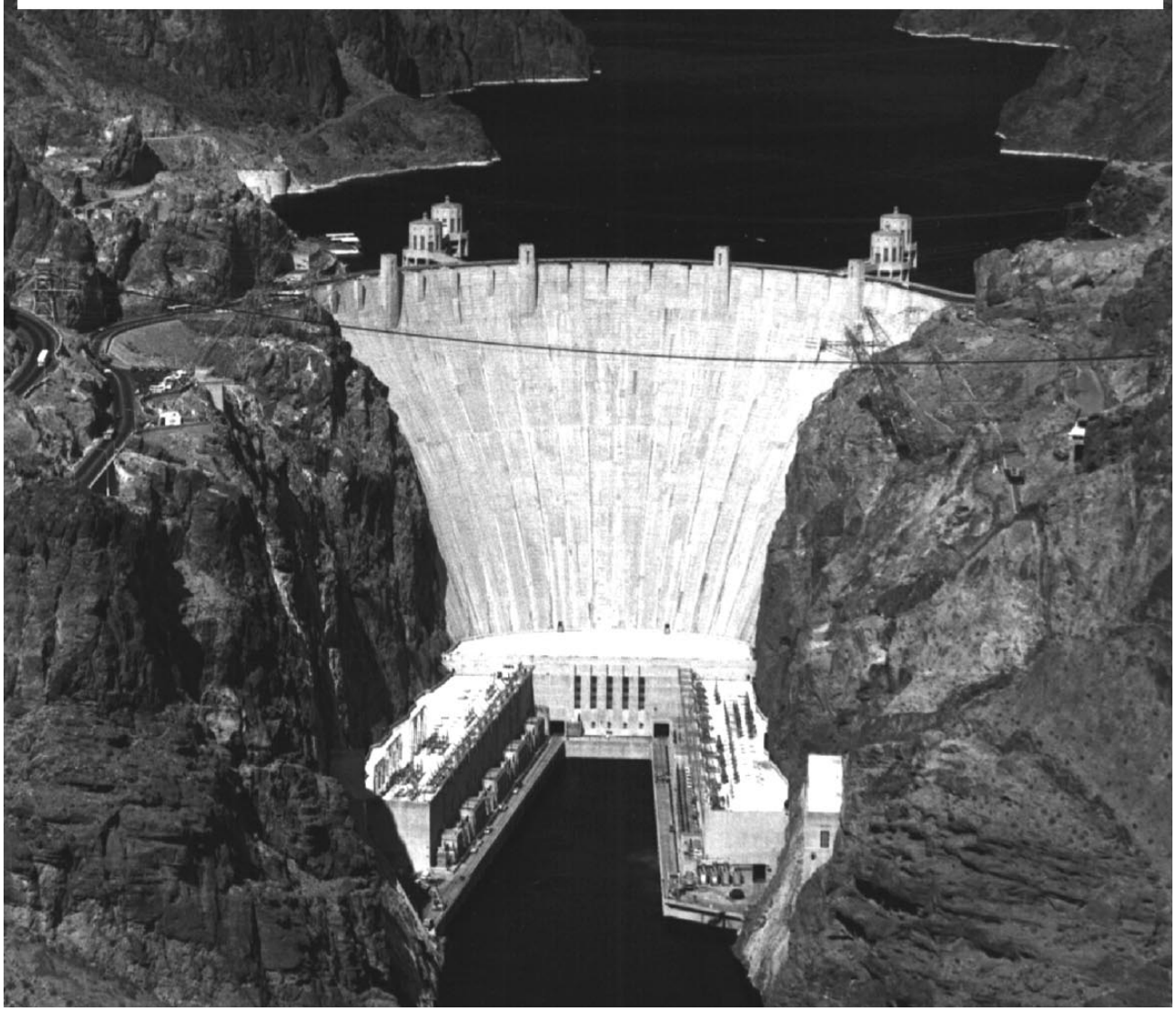
A common design used in the construction of dams is the arch dam. An arch dam is usually built in a narrow canyon and curves toward the water it contains. The canyon itself is used to support the dam. The force of the water against the dam is transferred outward to the canyon walls.

Purpose

In this lab, you will analyze the construction of an arch dam and find its surface area and volume. You will also study the fluid force on one of the dam's gates. You will use *Mathematica* to verify your results.

References

For more information about the calculus of dam design, see *Calculus, Understanding Change*, a three-part, half-hour video produced by COMAP and funded by the National Science Foundation.



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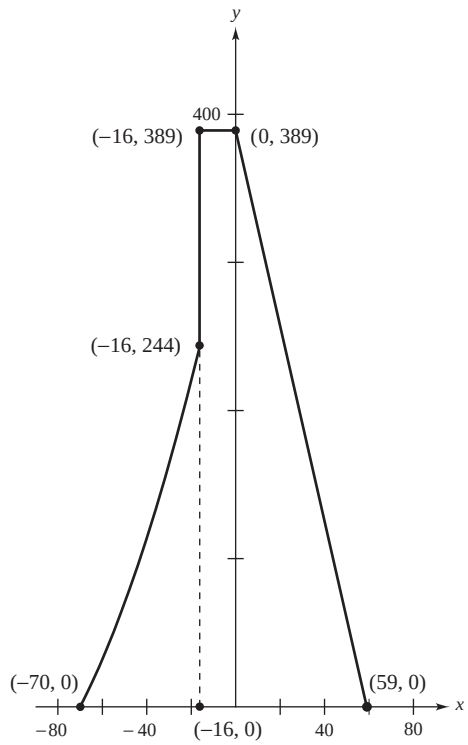
Data

One model for a cross section of an arch dam whose dimensions are given in feet is as follows.

$$f(x) = \begin{cases} 0.03x^2 + 7.1x + 350, & -70 \le x \le -16 \\ 389, & -16 < x < 0 \\ -6.593x + 389, & 0 \le x \le 59 \end{cases}$$

A graph of the cross section is given below.



The graph of a cross section of the arch dam is stored in the *Mathematica* file called LAB09.NB.

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Exercises

Name _____

Date _____ Class _____

Instructor _____

1. ***Calculating Area Without Calculus.*** Explain how you would calculate the area of a cross section of the arch dam without using Calculus. Use your explanation to calculate the area of a cross section of the arch dam. Is the area you found exact? Why or why not?

2. ***Calculating Area With Calculus.*** Explain how you would calculate the area of a cross section of the arch dam using Calculus. Use your explanation to calculate the area of a cross section of the arch dam. Is the area you found exact? Why or why not?

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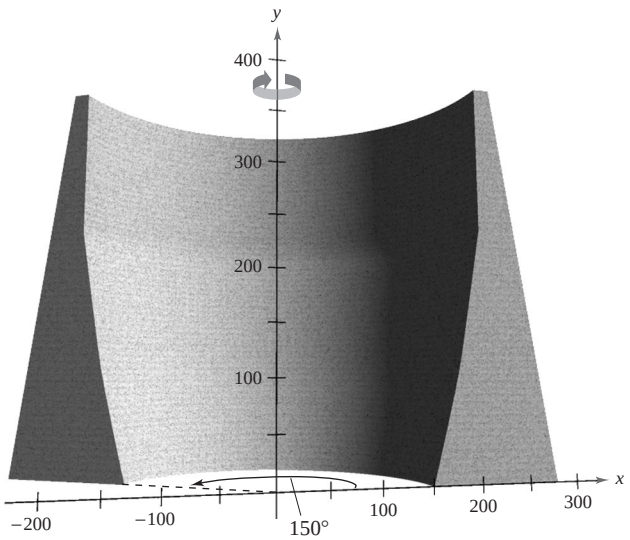
3. ***Calculating Area With Mathematica.*** Use *Mathematica* to calculate the area of a cross section of the arch dam. Is the value for area you found using *Mathematica* the same as the value you found in Exercise 2? If the values are not the same, explain why they are different.

4. ***Calculating Volume.*** To form an arch dam, the cross section described in this lab's Data is swung through an arc, rotating it about the *y*-axis as shown in the figure below. The cross section is rotated 150° and the axis of rotation is 150 feet. Explain how you would calculate the volume of the arch dam. Use your explanation to calculate the volume of the arch dam.



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5. ***Calculating Volume With Mathematica.*** Use *Mathematica* to calculate the volume of the arch dam. Is the volume you found using *Mathematica* the same as the volume you found in Exercise 4? If the values are not the same, explain why they are different.

Fluid Force on a Gate. **Use the following information to answer Exercises 6-9.** A series of vertical gates allows water from the dam to flow to an electrical plant. Each gate has an elliptical shape. So a gate can be properly designed, it is important to determine the fluid force that will be applied to the gate. The force F exerted by a fluid of constant weight-density w (per unit of volume) against a submerged vertical plane region from $y = c$ to $y = d$ is

$$F = w\int_c^d h(y)L(y)dy$$

where $h(y)$is the depth of the fluid at y and $L(y)$ is the horizontal length of the region at y. The top of the gate is 140 feet above the base of the dam, and the bottom of the gate is 112 feet above the base of the dam. The width of the gate at its center is 16 feet.

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6. Find the equation of the ellipse that describes the face of the gate. Assume that the origin of the coordinate system is located at the center of the gate. Use the equation of the ellipse

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

to find the horizontal length $L(y)$ of the region at y.

7. Find the equation that describes the depth $h(y)$ of the water at y. Assume that the origin of the coordinate system is located at the center of the gate and the water can reach the full height of the dam.

8. Find the integral for the fluid force on the gate. Assume the weight-density w of the water is 62.4 pounds per cubic foot. Evaluate the integral. In what unit of measure is the answer?

9. The gate is rotated 90°, so that the top of the gate is 134 feet above the base of the dam, and the bottom of the gate is 118 feet above the base of the dam. The width of the gate at its center is 28 feet. Will the pressure be more than, the same as, or less than the pressure you found in Exercise 8? Explain.

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