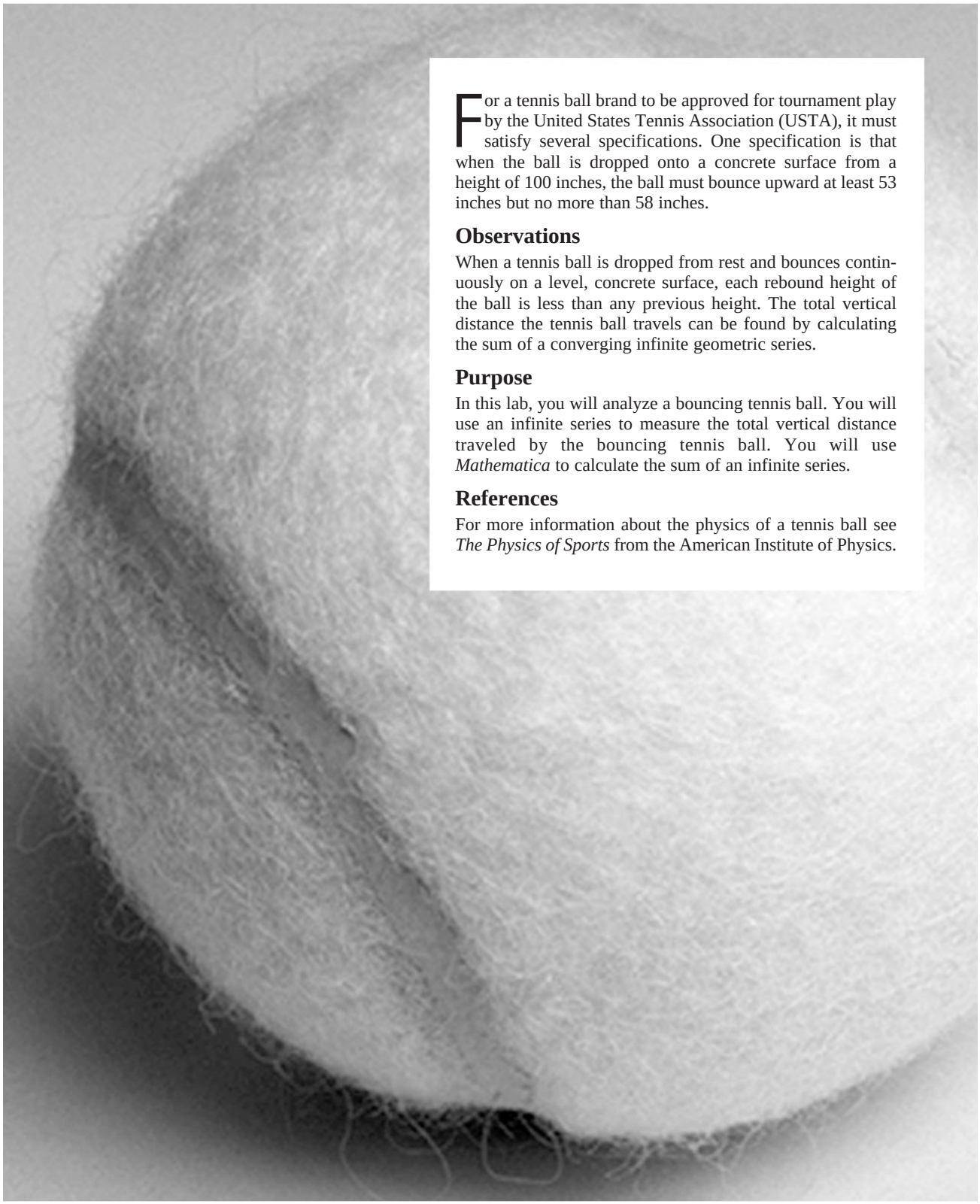
LAB 12

Analyzing a Bouncing Tennis Ball

Infinite Series



For a tennis ball brand to be approved for tournament play by the United States Tennis Association (USTA), it must satisfy several specifications. One specification is that when the ball is dropped onto a concrete surface from a height of 100 inches, the ball must bounce upward at least 53 inches but no more than 58 inches.

Observations

When a tennis ball is dropped from rest and bounces continuously on a level, concrete surface, each rebound height of the ball is less than any previous height. The total vertical distance the tennis ball travels can be found by calculating the sum of a converging infinite geometric series.

Purpose

In this lab, you will analyze a bouncing tennis ball. You will use an infinite series to measure the total vertical distance traveled by the bouncing tennis ball. You will use *Mathematica* to calculate the sum of an infinite series.

References

For more information about the physics of a tennis ball see *The Physics of Sports* from the American Institute of Physics.

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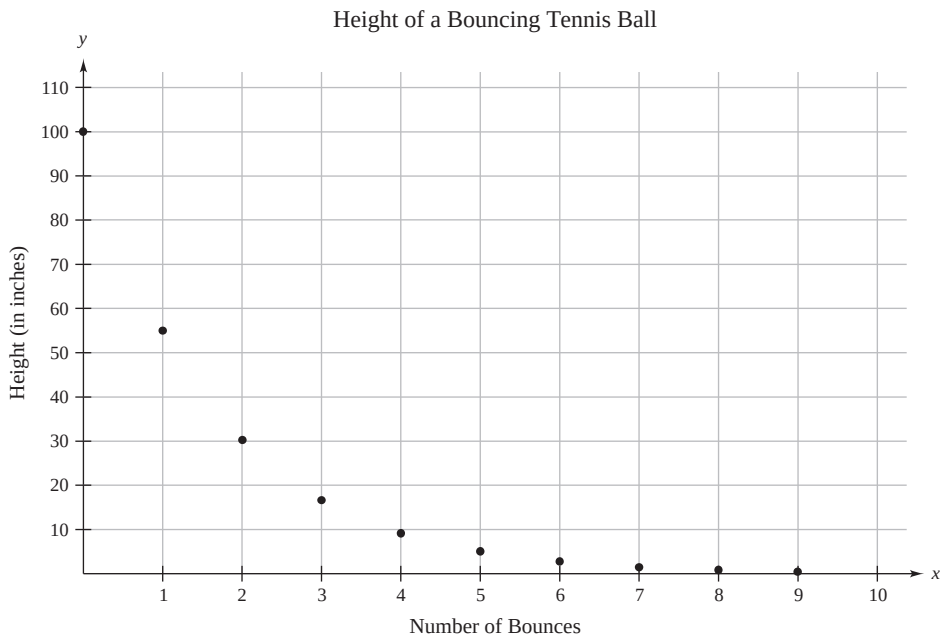
Data

One tennis ball that meets USTA specifications was dropped from a height of 100 inches onto a level, concrete surface. The height of the ball when it reached its apex after a bounce was recorded in the following table.

Number of bounces	0	1	2	3	4	5	6	7	8	9
Height (in inches)	100	55	30.25	16.64	9.15	5.03	2.77	1.52	0.84	0.46

A scatter plot of the data is given below.



The data in the table and the scatter plot are stored in the *Mathematica* file called LAB12.NB.

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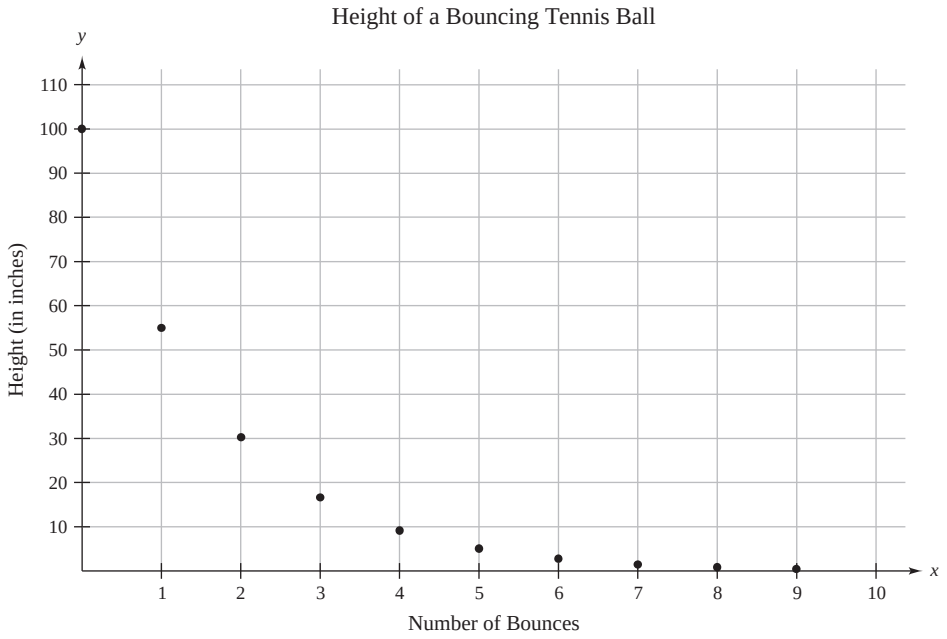
Exercises

Name _____

Date _____ Class _____

Instructor _____

1. ***Modeling the Data.*** What type of mathematical model do you think fits the bouncing tennis ball data? Explain. Apply your model to the bouncing tennis ball data. Then graph the model on the scatter plot below.



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2. ***Modeling the Data With Mathematica.*** An exponential equation is used in this lab's *Mathematica* file to model the bouncing tennis ball data. The equation is given by

$$y = y_0 p^n,$$

where y is the rebound height, y_0 is the initial height, p is the rebound rate, and n is the number of bounces. What value is used for y_0? What value is used for p? Explain how the value of p was determined. Compare this model to the one you used in Exercise 1. Is one model better than the other? Why or why not?

3. ***Still Bouncing?*** Do you think the tennis ball described in this lab's Data eventually came to a stop? Why or why not? Use the exponential model given in Exercise 2 to support your conclusion.

4. ***Finding the Total Vertical Distance Traveled.*** How would you find the total vertical distance traveled by the tennis ball described in this lab's Data? Describe a strategy to answer this question and use your strategy to determine the total vertical distance traveled by the tennis ball.

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5. ***Using a Geometric Series.*** A geometric series can be used to find the total vertical distance traveled by the tennis ball described in this lab's Data. The total vertical distance traveled is given by

$$D = D_0 + 2\sum_{n=1}^{\infty} D_0 p^n$$

where D is the total vertical distance, D_0 is the initial height, p is the rebound rate, and n is the number of bounces. How does this method of finding the total vertical distance traveled by the tennis ball compare to the one you used in Exercise 4? Let $D_0 = 100$ and $p = 0.55$ and calculate the value of D. Did you obtain the same answer as you did in Exercise 4? If not, explain why the answers are different.

6. ***Why Multiply by Two?*** Explain why the geometric series

$$2\sum_{n=1}^{\infty} D_0 p^n$$

is multiplied by 2 in the equation for D from Exercise 5.

7. ***A Legal Ball?*** A tennis ball was dropped from a height of 100 inches onto a level, concrete surface. After five bounces, the tennis ball had traveled a vertical distance of 376 inches. Is this a USTA sanctioned tennis ball? Explain how you determined your answer.

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8. ***Minimum and Maximum Distances.*** What is the minimum total vertical distance a USTA sanctioned tennis ball could travel if the tennis ball is dropped from a height of 100 inches onto a level, concrete surface? Under the same conditions, what is the greatest total vertical distance a USTA sanctioned tennis ball could travel?

9. ***Changing the Initial Height.*** The tennis ball described in this lab's Data is dropped onto a level, concrete surface again, this time from a height of 200 inches. The total vertical distance D traveled by the tennis ball has changed by what factor? Each time you multiply the initial height by a positive integer k, by what factor will you change D?

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