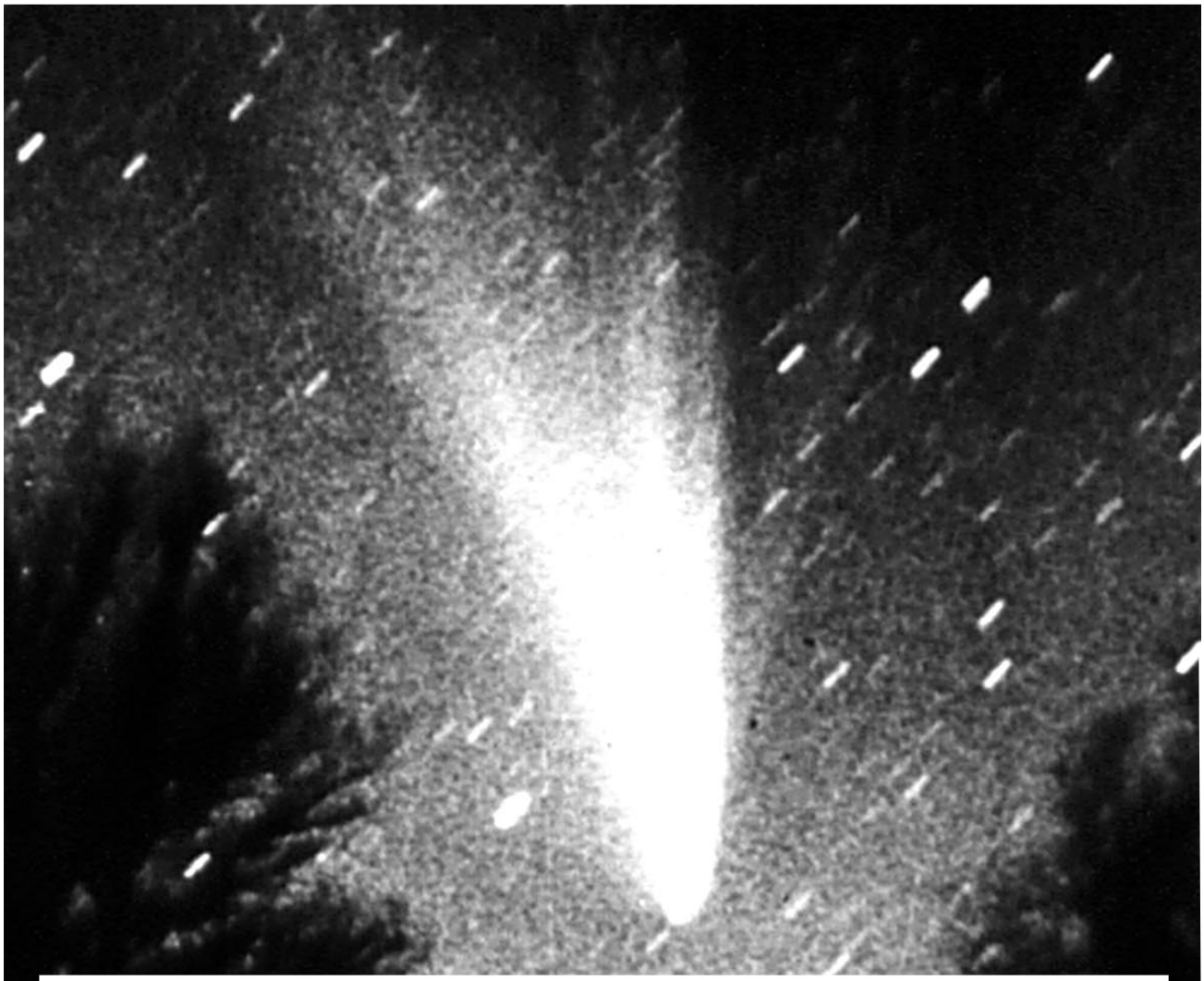
LAB 13

COMETS

Conics



With its high rate of activity and the ability to be seen by the naked-eye, Comet C/1995 O1 (Hale-Bopp), also referred to as Comet Hale-Bopp, was the cause of much excitement in early 1997. Discovered by Alan Hale and Thomas Bopp on July 23, 1995, it has been estimated that Comet Hale-Bopp last passed through the inner solar system over 4000 years ago.

Observations

The study of the orbits of Comet Hale-Bopp and other comets in our solar system leads to an interesting application of conics. The center of the sun is a focus of each orbit, and each orbit has a vertex at the point at which the comet is closest to the sun. Unlike comets with elliptical orbits, comets with parabolic or hyperbolic orbits pass through our solar system only once.

Purpose

In this lab, you will analyze the orbits of several comets and determine an equation for their orbits using rectangular and polar equations. You will use *Mathematica* to graph the equations of the orbits.

References

For more information about comets (plus asteroids and planets) see *Exploring Planetary Worlds* by David Morrison. Astronomical data about Comet Hale-Bopp courtesy of Daniel W. E. Green, Associate Director, Central Bureau for Astronomical Telegrams.

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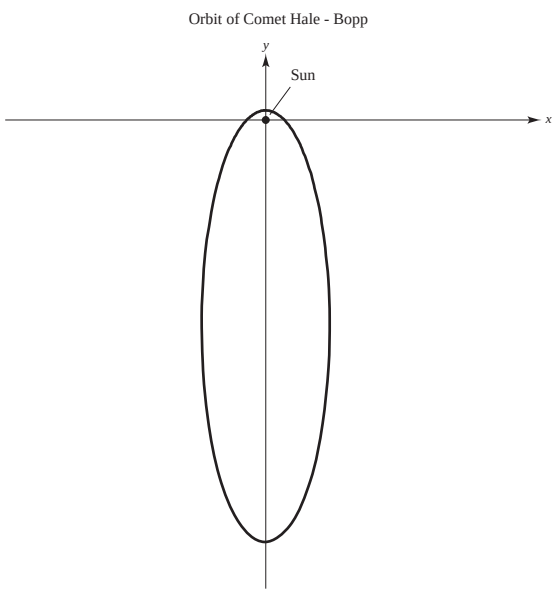
Data

Comet Hale-Bopp has an elliptical orbit with an eccentricity of $e \approx 0.995$. Comet Hale-Bopp's perihelion distance (the shortest distance to the sun) in 1997 was 0.914 astronomical unit (AU). (An astronomical unit is defined to be the mean distance between the earth and the sun, 92.956×10^6 million miles or 1.496×10^{11} meters.) A graphical representation of Comet Hale-Bopp's orbit is given below.



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Exercises

Name _____

Date _____ Class _____

Instructor _____

1. ***Finding a Polar Equation.*** Find a polar equation for Comet Hale-Bopp's orbit. Use an equation of the form

$$r = \frac{ed}{1 + e \sin \theta}$$

where e is the eccentricity and $|d|$ is the distance between the focus at the pole and its corresponding directrix. Use the equation to find Comet Hale-Bopp's aphelion distance (farthest distance from the sun). What value of θ corresponds to the aphelion distance? Explain your reasoning.

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2. ***Graphing a Polar Equation.*** Use *Mathematica* to graph the equation you found in Exercise 1 with θ varying from $\pi/2$ to $3\pi/2$. Then graph the equation again with θ varying from $\pi/2$ to $5\pi/2$, and again with θ varying from $\pi/2$ to $9\pi/2$. What do you observe?

3. ***A Set of Parametric Equations.*** Find a set of parametric equations for Comet Hale-Bopp's orbit with the sun at the origin. Use equations of the form

$$x = b\cos\theta + h \text{ and } y = a\sin\theta + k$$

where a is half of the length of the major axis, b is half of the length of the minor axis, and (h, k) is the center of the ellipse. Use *Mathematica* to graph the equations. Repeat Exercise 2 using the set of parametric equations and compare each graph to the corresponding polar graph.

4. ***Using a Rectangular Coordinate System.*** Find a rectangular equation for Comet Hale-Bopp's orbit. Use an equation of the form

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1.$$

(Note: Unlike the equations you found in Exercises 1 and 3, this equation does not place the sun at the origin.) Use *Mathematica* to graph the equation. Of the three different ways you have represented Comet Hale-Bopp's orbit in this lab, which way would you choose to represent the orbit? Explain your reasoning.

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5. ***Determining the Type of Orbit.*** The type of orbit for a comet can be determined as follows.

Ellipse: $v < \sqrt{\dfrac{2GM}{p}}$

Parabola: $v = \sqrt{\dfrac{2GM}{p}}$

Hyperbola: $v > \sqrt{\dfrac{2GM}{p}}$

In these three formulas, v is the velocity of the comet at the vertex (in meters per second), G is the gravitational constant where $G \approx 6.67 \times 10^{-11}$ cubic meters per kilogram-second squared, M is the mass of the sun where $M \approx 1.991 \times 10^{30}$ kilograms, and p is the distance between one vertex and one focus of the comet's orbit (in meters). Determine the value of

$$\sqrt{\frac{2GM}{p}}$$

for Comet Hale-Bopp. Show that the orbit of Comet Hale-Bopp is an ellipse if it was traveling at about 44 km/sec or less when it reached perihelion. If p is held constant, find a velocity at the perihelion that would mean Comet Hale-Bopp has a parabolic orbit. For what velocities at the perihelion would Comet Hale-Bopp have a hyperbolic orbit if p is held constant?

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6. ***Orbital Period of Comet Hale-Bopp.*** Estimates of Comet Hale-Bopp's orbit indicate that it last passed the earth around 2214 B.C (about 4210 years ago) and it will return around 4377 A.D. (about 2380 years from now). List several factors that could be changing the orbital period of Comet Hale-Bopp.

7. ***Applying Kepler's Second Law to Comet Hale-Bopp.*** Kepler's Second Law states that as a planet moves about the sun, a ray from the sun to the planet sweeps out equal areas in equal times. This law can also be applied to comets (or other heavenly bodies) with elliptical orbits. Applying Kepler's Second Law to Comet Hale-Bopp, what conclusion can you make about the comet's velocity as it approaches perihelion distance? What conclusion can you make about the comet's velocity as it approaches aphelion?

8. ***Verifying Kepler's Second Law.*** The length of time for Comet Hale-Bopp to move in its orbit from $\theta = \alpha$ to $\theta = \beta$ is given by

$$\frac{t}{2380} = \frac{\text{Area of Segment}}{\text{Area of Ellipse}} = \frac{\frac{1}{2}\int_{\alpha}^{\beta} r^2\, d\theta}{\pi ab}$$

where t is the time in years. Use *Mathematica* to calculate the number of days t it takes for the comet to move from

$$\theta = \frac{3\pi}{4} \quad \text{to } \theta = \frac{5\pi}{4}.$$

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