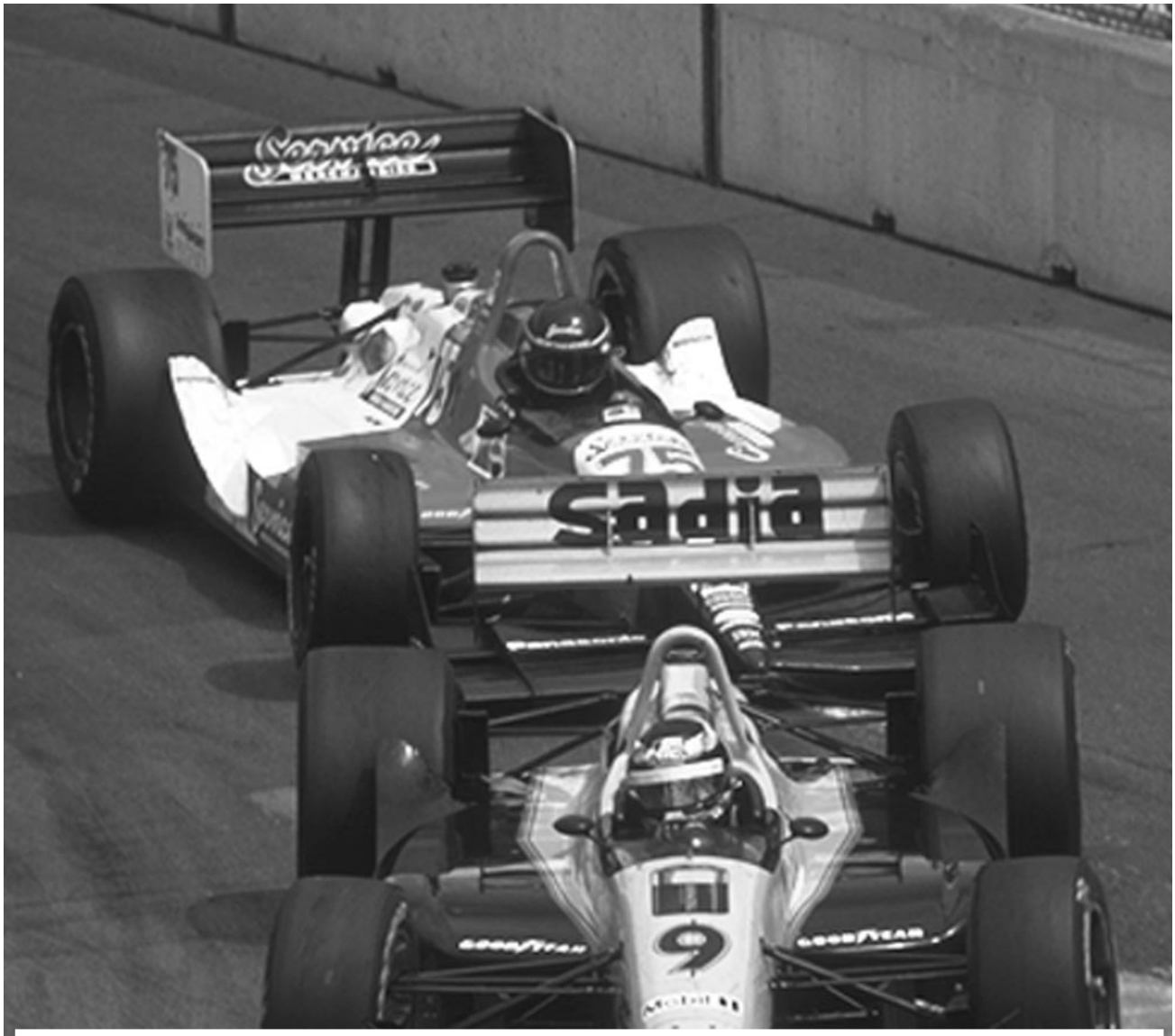
LAB 15

Race-Car Cornering

Vector-Valued Functions



The Indianapolis Motor Speedway is the host of the internationally famous Indianapolis 500. The speedway has two 3300-foot-long straightaways, two 660-foot-long straightaways, and four 1320-foot-long turns. Not all speedways have this symmetry, and the lack of it can cause variation in the speed and velocity with which a race car can negotiate each turn.

Observations

The Walt Disney World Speedway in Orlando, Florida has three straightaways and three turns. The straightaways have lengths of 1487 feet, 572 feet, and 909 feet. The turns have lengths of 990.35 feet, 976.35 feet, and 417.40 feet.

Purpose

In this lab, you will analyze the turns at Walt Disney World Speedway using vector-valued functions. You will use *Mathematica* to aid you in your analysis.

References

Information about Indianapolis Motor Speedway and Walt Disney World Speedway courtesy of Kevin Forbes, Director of Engineering & Construction at Indianapolis Motor Speedway.

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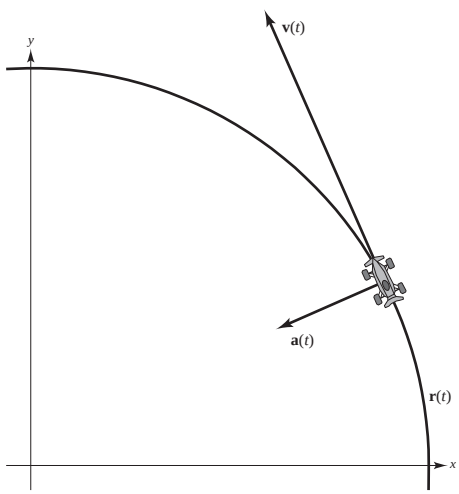
Data

The position of a certain race car on Turn 1 at Walt Disney World Speedway can be approximated by the following position vector, where t is the time measured in seconds.

$$\begin{aligned}\mathbf{r}(t) &= x(t)\mathbf{i} + y(t)\mathbf{j} \\ &= 446.85\cos(0.4759t)\mathbf{i} + 446.85\sin(0.4759t)\mathbf{j}, \qquad 0 < t < 4.66\end{aligned}$$

The race car's velocity (comprising both speed and direction) can be represented by a velocity vector. The race car's acceleration (any change in velocity, whether in speed or direction) can be represented by an acceleration vector.



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Exercises

Name _____

Date _____ Class _____

Instructor _____

1. ***Finding Velocity and Acceleration Vectors.*** Find the velocity and acceleration vectors for the race car whose position vector is given in this lab's Data. What is the speed of the race car as it moves through the turn? Is the speed of the race car through the turn constant? Is the velocity of the race car constant? What is the acceleration of the race car as it moves through the curve? Is the acceleration vector **a** constant? Explain your reasoning.

2. ***Turn 2.*** Repeat Exercise 1 for Turn 2, where the position vector is given by

$$\mathbf{r}(t) = 522.47\cos(0.4211t)\mathbf{i} + 522.47\sin(0.4211t)\mathbf{j}, \qquad 0 < t < 4.44.$$

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3. ***Turn 3.*** Repeat Exercise 1 for Turn 3, where the position vector is given by

$$\mathbf{r}(t) = 733.89\cos(0.3797t)\mathbf{i} + 733.89\sin(0.3797t)\mathbf{j}, \qquad 0 < t < 1.50.$$

4. ***Taking Turns.*** Use *Mathematica* to determine the curvature of each turn and record your results below.

Curvature of Turn 1:

Curvature of Turn 2:

Curvature of Turn 3:

Considering each turn's curvature and the race car's velocity vector for each turn, which turn do you think is the "easiest" for the race car to negotiate? Which turn do you think is the "hardest" for the race car to negotiate? Explain your reasoning.

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5. ***Make a Table.*** Use the position vector given in this lab's Data to make a table of the race car's positions for several values of t between $t = 0.00$ and $t = 4.66$.

t	0.00	1.00	2.00	3.00	4.00	4.66
$x(t)$						
$y(t)$						

6. ***Graphing Velocity and Acceleration Vectors.*** Plot the results of Exercise 5 on the graph below. At each position, draw the velocity vector and the acceleration vector. What does each velocity vector have in common? What does each acceleration vector have in common?

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7. ***Staying on Track.*** Centripetal force is the force required to hold a moving object in a circular path. Centripetal force can be calculated using the formula

$$F = \frac{mv^2}{r}$$

where F is the centripetal force, m is mass (in pounds), v is speed (in feet per second), and r is radius (in feet). Assume the mass of the race car described in this lab's Data is 1800 pounds. Find the centripetal force required in each turn to hold the race car on the track.

Turn 1:

Turn 2:

Turn 3:

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