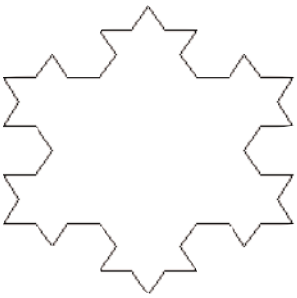
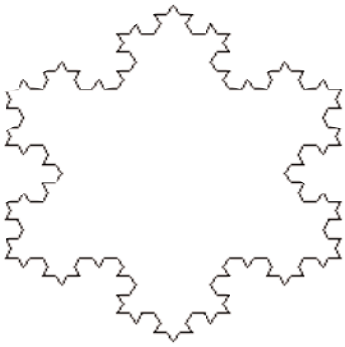
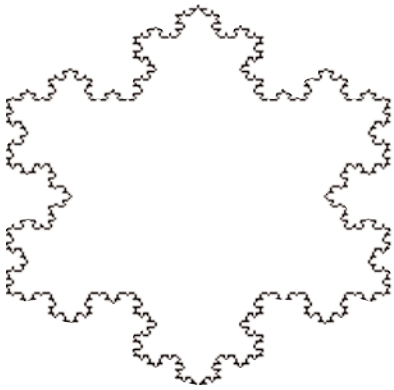
LAB 11

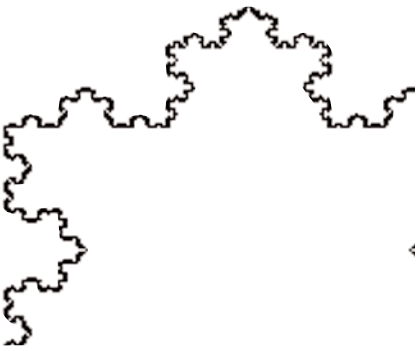
# KOCH SNOWFLAKE

## Fractals









Clouds, biological growth, and coastlines are examples of real-life phenomena that seem too complex to be described using typical mathematical functions or relationships. By developing fractal geometry, Benoit Mandelbrot (1924–) found a way to describe real-life phenomena using a new mathematical construct.

### Observations

Mandelbrot first used the word fractal in 1975 to describe any geometric object whose detail is not lost as it is magnified. Being "endlessly magnifiable" means that you can zoom in on any portion of the fractal and it will be identical to the original fractal.

### Purpose

In this lab, you will analyze some fractals. You will use *Mathematica* to "zoom in" on different parts of a fractal.

### References

For more information about fractals see *The Fractal Geometry of Nature* by Benoit Mandelbrot.

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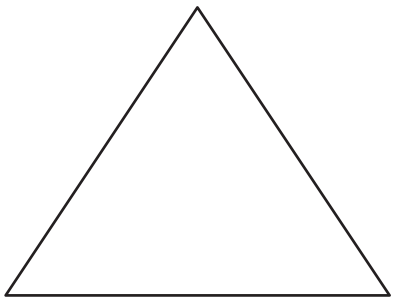
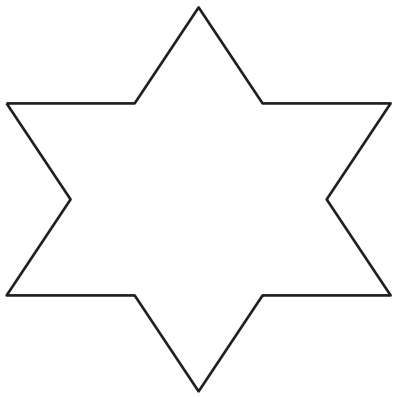
## Data

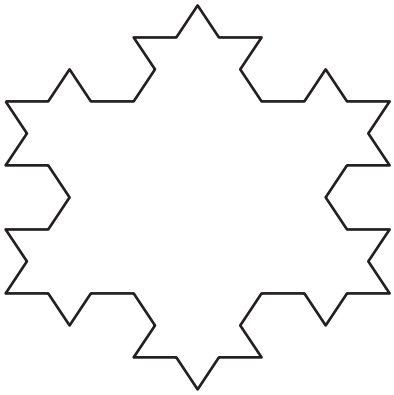
One of the "classic" fractals is the Koch snowflake, named after Swedish mathematician Helge von Koch (1870–1924). The construction of the Koch snowflake begins with an equilateral triangle whose sides are one unit long. In the first iteration, a triangle with sides one-third unit long is added in the center of each side of the original. In the second iteration, a triangle with sides one-ninth unit long is added in the center of each side of the first iteration. Successive iterations continue this process indefinitely. The first four stages are shown below.



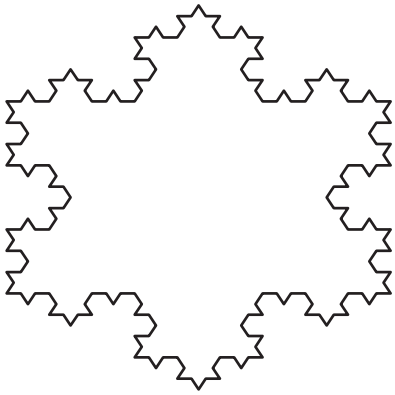
Stage 0



Stage 1



Stage 2



Stage 3

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# Exercises

Name _____

Date _____ Class _____

Instructor _____

1. ***Complete the Table.*** Use the iteration for the Koch snowflake as described in this lab's Data to complete the following table.

| Stage | Sides | Perimeter | Area |
|---|---|---|---|
| 0 | 3 | 3 | $\frac{\sqrt{3}}{4}$ |
| 1 | 12 | 4 | $\frac{\sqrt{3}}{3}$ |
| 2 | | | |
| 3 | | | |
| 4 | | | |
| 5 | | | |
| ⋮ | | | |
| $n$ | | | |

2. ***Perimeter and Area.*** Using the results of Exercise 1, do you think it's possible for a closed region in the plane to have a finite area and an infinite perimeter? Explain your reasoning.

_____

_____

_____

_____

_____

_____

_____

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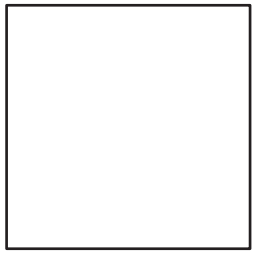
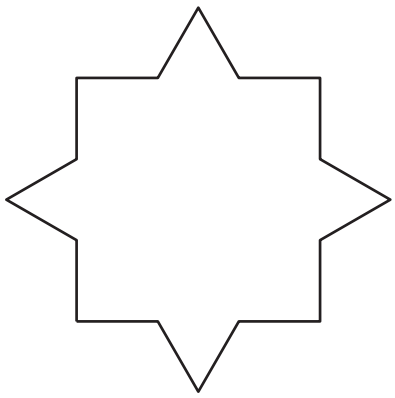
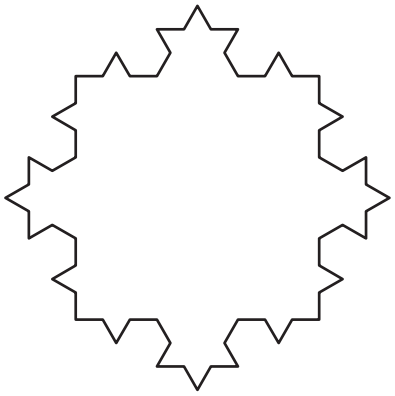
**3.** ***Start with a Square.*** A variation of the Koch snowflake is shown below. This time, the construction begins with a square whose sides are one unit long. In the first iteration, a triangle with sides one-third unit long is added in the center of each side of the square. In the second iteration, a triangle with sides one-ninth unit long is added in the center of each side. Successive iterations continue this process indefinitely. The first four stages are shown below.



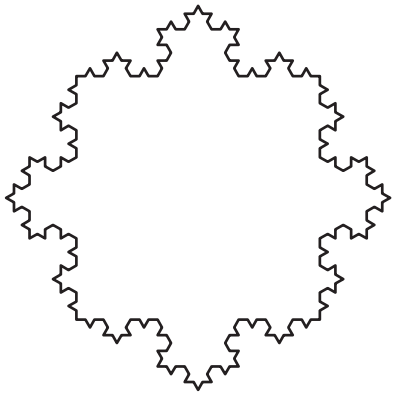
Stage 0



Stage 1



Stage 2



Stage 3

| Stage | Sides | Perimeter | Area |
|---|---|---|---|
| 0 | 4 | 4 | 1 |
| 1 | 16 | $\frac{16}{3}$ | $1 + \frac{\sqrt{3}}{9}$ |
| 2 | | | |
| 3 | | | |
| 4 | | | |
| 5 | | | |
| $\vdots$ | | | |
| $n$ | | | |

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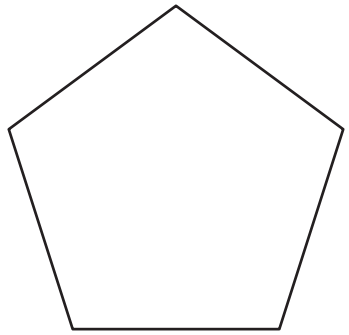
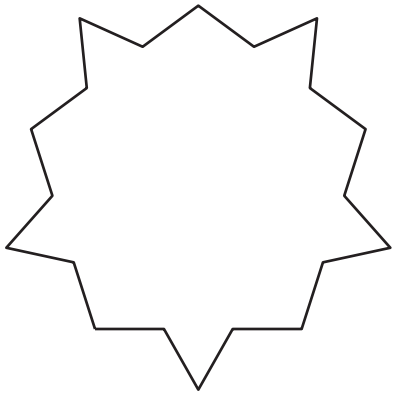
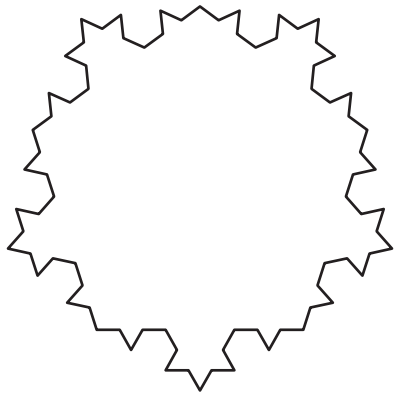
**4.** ***Start with a Pentagon.*** Another variation of the Koch snowflake is shown below. This time, the construction begins with a pentagon whose sides are one unit long. In the first iteration, a triangle with sides one-third unit long is added in the center of each side of the pentagon. In the second iteration, a triangle with sides one-ninth unit long is added in the center of each side. Successive iterations continue this process indefinitely. The first four stages are shown below.



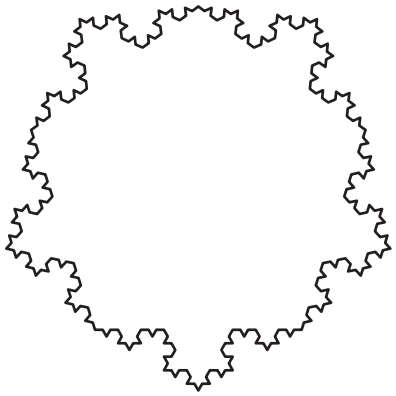
Stage 0



Stage 1



Stage 2



Stage 3

How many sides will the *n*th iteration have? Do you think the Koch snowflake described in this exercise will have an area equal to, greater than, or smaller than the area of the Koch snowflake described in Exercise 1? Do you think its area will be equal to, greater than, or smaller than the area of the Koch snowflake described in Exercise 3? Explain your reasoning.

**5.** ***Comparing Perimeters.*** Do you think the Koch snowflake described in Exercise 4 will have a perimeter equal to, greater than, or smaller than the perimeter of the Koch snowflake described in Exercise 1? Do you think its perimeter will be equal to, greater than, or smaller than the perimeter of the Koch snowflake described in Exercise 3? Explain your reasoning.

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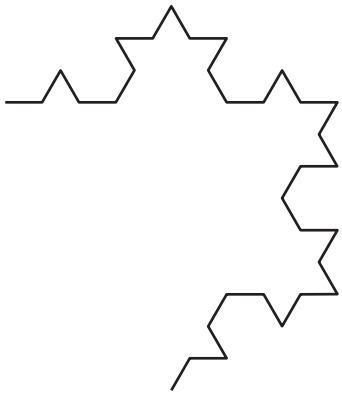
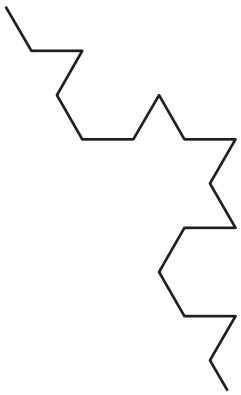
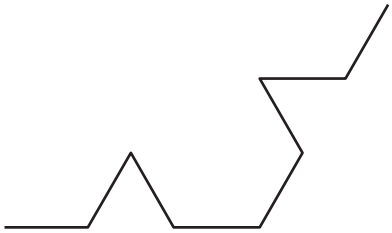
**6.** ***A Koch Curve Using Mathematica.*** *Mathematica* was used to generate the following graphs of a Koch curve. Use *Mathematica* to reproduce each graph and record the viewing window you used. Can you find more than one viewing window that reproduces each graph below? If so, list the viewing window values *xmin*, *xmax*, *ymin*, and *ymax* you used for each graph.







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