LAB 4

Boyle's Law

Differentiation

Robert Boyle (1627–1691), who is often considered the founder of modern chemistry, made studies of acids and bases, the calcination of metals, combustion, the nature of colors, and the propagation of sound. Boyle is best known for his experiments on gases and the formulation of the gas law that bears his name.

Observations

A unique property of a gas is that it is compressible. A given volume of a gas, unlike a solid or a liquid, can be squeezed into a smaller container or compressed by the application of pressure. As the volume of the gas decreases, the pressure that the gas exerts on its container increases. Boyle determined, after many experiments, that the volume of a sample of gas at a given temperature varies inversely with the applied pressure.

Purpose

In this lab, you will explore the relationship between the pressure of a trapped gas and its container. You will use *Mathematica* to aid you in your exploration.

References

For more information about Robert Boyle, *The Life of the Honourable Robert Boyle* by R. E. W. Maddison.

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Data

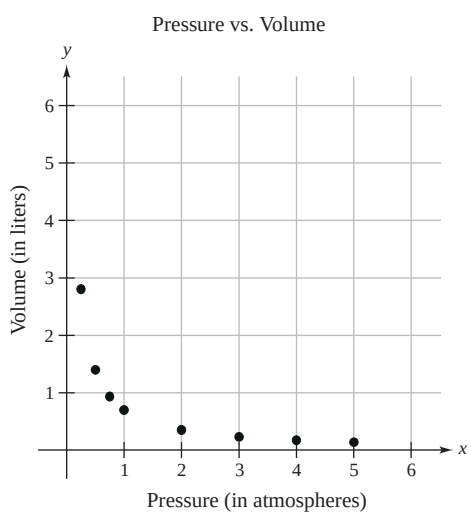
Boyle's Law states that the volume of a sample of gas at a given temperature varies inversely with the applied pressure. Mathematically, Boyle's Law can be written as

$$PV = k \text{ or } V = \frac{k}{P}$$

where V is the volume of the gas, P is the applied pressure, and k is a constant. Boyle's Law is only valid for systems which are at a constant temperature and contain a constant mass of gas. When these conditions are met, the product of the pressure and the volume remains nearly constant. The pressure-volume data in the table below is for 1.000 g of oxygen at 0°C.

Pressure, *P* (atm)	**Volume, *V* (L)**
0.25	2.801
0.50	1.400
0.75	0.9333
1.00	0.6998
2.00	0.3495
3.00	0.2328
4.00	0.1744
5.00	0.1394

A scatter plot of the data is given below.



The data in the table and the scatter plot is stored in the *Mathematica* file called LAB04.NB.

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Exercises

Name _____

Date _____ Class _____

Instructor _____

1. ***Complete the Table.*** Calculate the product of the pressure and the volume (PV) for the pressure-volume data discussed in this lab's Data and complete the table below.

Pressure, P (atm)	Volume, V (L)	PV
0.25	2.801	
0.50	1.400	
0.75	0.9333	
1.00	0.6998	
2.00	0.3495	
3.00	0.2328	
4.00	0.1744	
5.00	0.1394	

Is the product of the pressure and the volume approximately equal to a constant number k? If so, determine the value of the constant k and use *Mathematica* to graph the equation

$$V = \frac{k}{P}$$

with the pressure-volume data. Use the graph to verbally describe the relationship between the pressure of a collected gas (in this case, oxygen) and its volume. Is the pressure ever zero?

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2. ***Pressure-Volume Relationship.*** Use *Mathematica* to graph the pressure-volume data given in this lab's Data with the volume on the vertical axis and $1/P$ on the horizontal axis. Is the data nearly linear? If so, determine the slope of a line that approximates the data. Verbally describe what the slope means in the context of the data. What is the y-intercept of the line? What does the y-intercept mean in the context of the data?

3. ***Using the Derivative.*** The equation for Boyle's Law is

$$PV = k.$$

Solve the equation for V and use the derivative to show that the rate of change of the volume with respect to pressure is inversely proportional to the square of the pressure.

4. ***Graphing the Derivative.*** Use the result of Exercise 3 to find the derivative dV/dP of

$$V = \frac{k}{P}$$

where k is the value you determined in Exercise 1. Then use *Mathematica* to graph

$$V = \frac{k}{P}$$

and its derivative on the same coordinate axes. Is the graph of the derivative always negative? Why or why not? What is the relationship between the slope of the derivative's tangent line and the graph of

$$V = \frac{k}{P}?$$

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5. ***Graphical Estimation.*** Use the graph in this lab's Data to estimate the missing volume data in the following table.

Pressure, *P* (atm)	Volume, *V* (L)
1.00	0.6998
1.50	
2.00	0.3495
2.50	
3.00	0.2328
3.50	
4.00	0.1744
4.50	

6. ***Proof.*** Boyle's Law can be used to calculate the volume occupied by a gas when the applied pressure changes. Let P_1 and V_1 be the initial pressure and volume, respectively, of a gas. After a change in pressure from P_1 to P_2, show that the new volume V_2 can be found by the following equation.

$$V_2 = \frac{P_1 V_1}{P_2}$$

7. ***Missing Data.*** Use the equation given in Exercise 6 to find the missing volume data in the following table.

Pressure, *P* (atm)	Volume, *V* (L)
1.00	0.6998
1.50	
2.00	0.3495
2.50	
3.00	0.2328
3.50	
4.00	0.1744
4.50	

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8. ***Charles's Law.*** In this lab's Data, we stated that Boyle's Law is only valid for systems which are at a constant temperature. Jacques Charles (1746–1823) discovered that the volume of a sample of gas at a constant pressure increases linearly with temperature and can be written as

$$V = mT$$

where V is the volume of the gas, T is the temperature in Kelvins, and m is a constant. After a change in temperature from T_1 to T_2, show that the new volume V_2 can be found by the following equation.

$$V_2 = \frac{T_2 V_1}{T_1}$$

9. ***Combining Boyle's Law and Charles's Law.*** Boyle's Law and Charles's Law can be combined and expressed in a single statement: The volume occupied by a sample of gas is proportional to the temperature (in Kelvins) divided by the pressure and can be written as

$$V = k\frac{T}{P}$$

where V is the volume of the gas, T is the temperature in Kelvins, P is the applied pressure, and k is a constant. After a change in temperature from T_1 to T_2 and a change in pressure from P_1 to P_2, show that the new volume V_2 can be found by the following equation.

$$V_2 = \frac{P_1 T_2 V_1}{P_2 T_1}$$

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