LAB 17

Satellite Dishes, Flashlights, and Solar Energy Collectors

Using the Reflective Property of Parabolas

Many objects take advantage of simple, yet powerful, mathematical properties. For example, a parabolic mirror used by a solar energy collector takes advantage of a special property of parabolas. This special property of parabolas is also used in more common objects, such as flashlights and satellite receiving dishes.

Observations

One way to describe the property is to say that *all incoming rays that are parallel to the axis of the parabola reflect directly through the focus of the parabola*. Likewise, *all outgoing rays from the focus that come in contact with the parabola will be reflected parallel to the axis of the parabola*. This property of parabolas is sometimes called the reflective property of parabolas.

Purpose

In this lab, you will analyze the use of parabolas and the reflective property of parabolas in satellite receiving dishes, solar energy collectors and flashlights. You will also analyze the advantages of other geometrical shapes. You will use *Mathematica* to aid you in your analysis.

References

For more information about the satellite television industry, see "Satellite TV Comes Down to Earth", *Popular Science*, November, 1996.

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Data

The shape of a satellite dish is a circular paraboloid. One way to model a circular paraboloid is by using a special case of the elliptic paraboloid formula

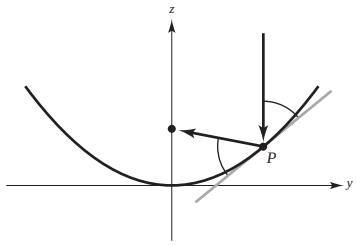
$$z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$$

and using $a = b$ to write the formula as

$$z = \frac{x^2}{a^2} + \frac{y^2}{a^2}.$$

For this model, the paraboloid opens up and has its vertex at the origin. For a satellite dish, the axis of the paraboloid should pass through the satellite so that all incoming signals are parallel to the paraboloid's axis. When incoming rays strike the surface of the paraboloid at a point P, they reflect at the same angle as they would if they were reflecting off a plane that was tangent to the surface at point P, as shown in the left-hand figure below.

In the plane, all parabolas have a special reflective property. One way to describe the property is to say that *all incoming rays that are parallel to the axis of the parabola reflect directly through the focus of the parabola*, as shown in the right-hand figure below. In a similar manner, all rays coming into a paraboloid are reflected to the paraboloid's focus.



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Exercises

Name _____

Date _____ Class _____

Instructor _____

1. ***Where is the Receiver?*** You are designing a satellite receiving dish. At what point should you place the receiver? Explain your reasoning.

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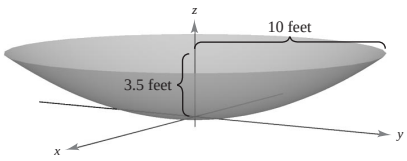
2. ***Find an Equation.*** Determine an equation of the form

$$z = \frac{x^2}{a^2} + \frac{y^2}{a^2}$$

for a satellite dish that has a radius of 10 feet and a depth of 3.5 feet as shown in the figure below.



What is the domain and range of the equation? At what point should you place the receiver?

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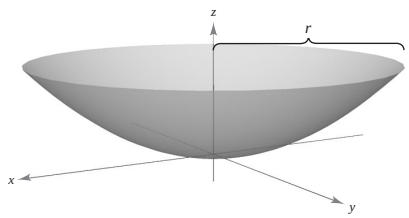
3. ***Using a Surface of Revolution.*** Imagine that the parabola given by

$$x^2 = 4py$$

is revolved about the y-axis to form a dish of radius r (see figure below).



This surface of revolution can be used as a model for a satellite receiving dish. If $p = 5$, then the parabola is

$$x^2 = 20y$$

and has its vertex at the origin and focus at (0, 5). Part of the cost of constructing a receiving dish is related to surface area. Find the surface area of the dish formed by revolving a segment of the parabola $x^2 = 20y$ about the y-axis to form a dish of radius r. The surface area is roughly proportional to the cube of what factor?

4. ***Comparing Answers.*** Use *Mathematica* to find the surface area of the dish formed by revolving a segment of the parabola $x^2 = 20y$ about the y-axis to form a dish of radius r. Compare the surface area you found in Exercise 3 to *Mathematica*'s value and verify that the answers are equivalent. If the answers are not equivalent, explain why they are different.

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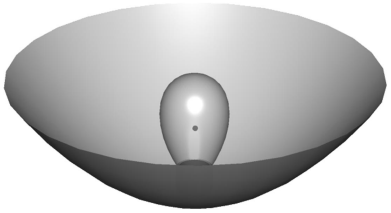
5. ***Using an Elliptic Paraboloid as a Reflector.*** Instead of being used as a receiver like a satellite dish, a flashlight uses an elliptic paraboloid as a reflector. The flashlight's bulb is placed at the focus of a parabolic reflector. A certain flashlight has a reflector that is 8 centimeters wide at the head of the flashlight. The flashlight's bulb is located at the focus of the parabola, 1.5 centimeters from the vertex of the reflector as shown in the figure below.



Find an equation for a cross section of the reflector of the form

$$x^2 = 4py$$

where x and y are measured in centimeters. Determine a segment of the parabola that can be revolved about the y-axis to form the reflector. Find the surface area of the reflector formed by the surface of revolution.

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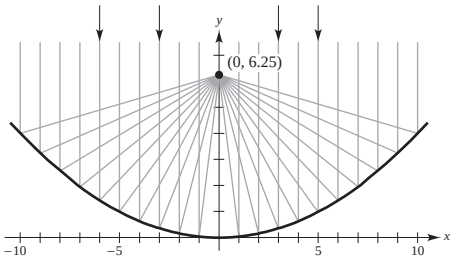
6. ***A Parabolic Solar Collector.*** The world's largest solar-thermal complex, Luz International, is located in California's Mojave Desert. Parabolic mirrors that look like troughs are used to reflect the sun's rays onto tubes filled with oil. The heated oil is then used to boil water, which sends steam to a turbine. Mechanical drives slowly rotate the mirrors to keep the reflected sunlight focused on the oil-filled tubes.

A cross section of a parabolic mirror used in this type of parabolic solar collector can be modeled by

$$y = \frac{x^2}{25}$$

where x and y are measured in feet. The oil-filled tube is located at the focus of the parabola as shown in the figure below



Describe the graph of the collector with respect to three dimensions. If the collector is 18 feet long, determine the surface area of the collector and the equation of the focal line.

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