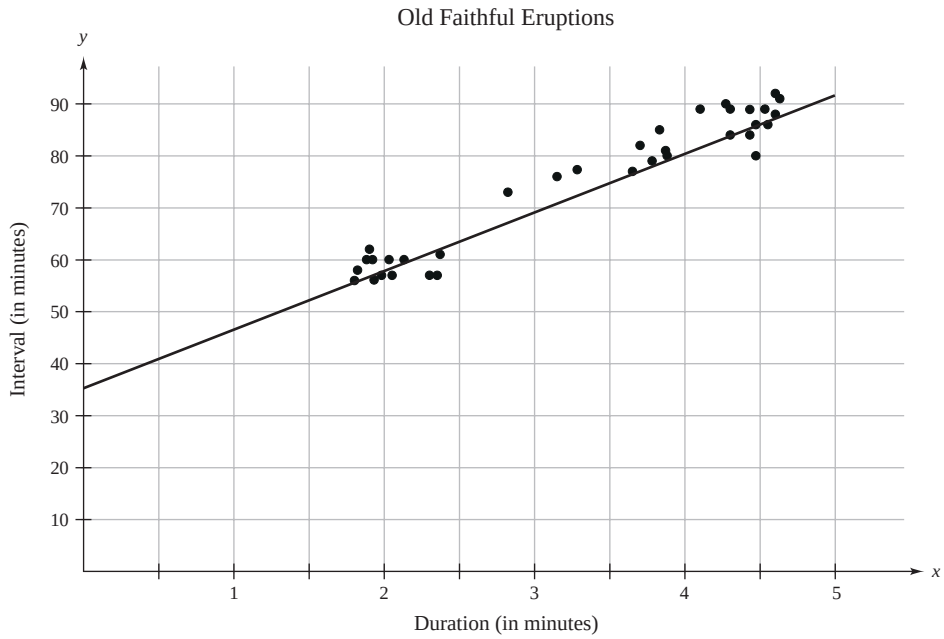
ANSWER KEY FOR LAB MANUALS

LAB 1 MODELING OLD FAITHFUL'S ERUPTIONS

Modeling Data

1. An acceptable answer should state that the duration x and interval y have a positive correlation. This means the student should understand that as the duration of an eruption increases, the interval between eruptions increases. Likewise, as the duration of an eruption decreases, the interval between eruptions decreases. A reasonable explanation of why x and y are related in this manner is that after an eruption, water seeps into the geyser's plumbing system to replace the water that was forced out by the eruption. For a long eruption, more water is forced out than a short eruption and it will take longer for the plumbing system to refill with water. Also, it will take longer for the water to be heated to produce the next eruption.

2. A best-fitting line drawn by hand should be reasonably similar to the one shown in the graph below.



The equation given should match the graph. For the graph shown above, the equation is

$$y = 11.824x + 35.301.$$

3. A conclusion about how good a model is will vary. However, the reasoning to support the conclusion should be sound. A discussion of how closely the model fits the data graphically and numerically, perhaps giving the model's average percent error and other valid mathematical reasoning would be appropriate.

4. A better-fitting linear model would have a smaller sum because the square of the differences between the actual y-values and the model's values would be smaller than the differences between the actual y-values and the values of the model from Exercise 2. Because the better-fitting linear model has smaller differences between the actual y-values and the model's values, the sum of the square of these differences will be smaller.

5. Squaring the differences prevents negative differences from making the total difference look smaller than it is.

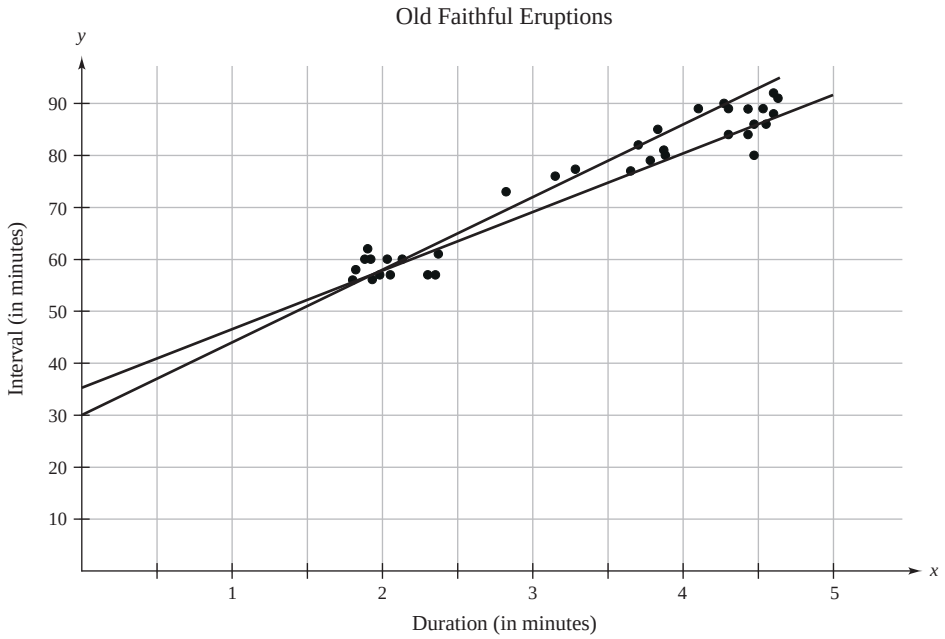
6. Students should choose the model that results in the smallest sum of the squared differences of the actual y-values and the model's y-values. This is the model that fits the data given in this lab "better." Some students may pick the Yellowstone Park model just because it is the Yellowstone Park model, even if the student's model is a "better" fit to the data given in this lab. If that is the case, check their reasoning. Students that choose the Yellowstone Park model may decide that the model used by Yellowstone Park to predict the intervals between Old Faithful's eruptions is based on a much larger set of data than what is given in this lab and therefore is more accurate in the long term.

7. The least squares model should be the best fit to this lab's data because it minimizes the differences between its results and the data.

8. *Model from Exercise 2:* Answer will vary

Yellowstone Park Model: $y = 14x + 30$

Least Squares Model: $y = 11.824x + 35.301$



9. A quadratic model is a slightly better fit. Students may site a lower least squares difference or a slight downward curve in the visual pattern of the data.

LAB 2 THE LIMIT OF SWIMMING SPEED

Finding Limits

1. Answers will vary. Check the reasonableness of the answers. If a student thinks there is no limit to human athletic performance, they should explain how an athlete can swim 100 meters in 0 seconds (or better).

2. Estimates of a lower limit should range between 45–48 seconds. An estimation for the men's 100-meter freestyle record in the year 2000 should be about 48 seconds.

3. Using graphical estimation to predict a reasonable record for a man to swim 100 meters in the year 2000, an answer should be about 47.5 seconds. A numerical estimate can be obtained by substituting $x = 100$ into the model to obtain $y \approx 47.7435$.

4. The estimate for Exercise 3 should be the same as the one obtained by *Derive*, *Maple*, *Mathcad*, *Mathematica*, and a graphing utility. Answers will vary. Some students may reason that the estimate in Exercise 2 is the more reasonable estimate because the data varies from the model. Other students may reason that the estimate in Exercises 3 and 4 is the more reasonable estimate because they are more exact.

5. The y-values given in the following table have been rounded to four decimal places.

Year, x	100	150	200	250	300
Time, y	47.7435	45.7794	45.1112	44.8114	44.6538

The lower limit appears to be about 44.6 seconds.

6. The model is best for the years given in the lab's Data. The model is not as reliable for years outside this range. For example, the record in 1905 was 65.8 seconds, but the model says the record was about 1277.9 seconds. Likewise, the model has no way of accounting for future trends, such as improved training methods, better nutrition, better athletes, etc.

7. Answers will vary. Logic tells us there must be a lower limit and the limit must not lie below zero. The lab, however, only shows that there is a model, covering 30 years of data, with a lower limit.

LAB 3 FALLING OBJECTS

First and Second Derivatives

1. A quadratic model would be a good fit to the height data. A linear model would be a good fit to the velocity data. They are both decreasing.

2. The values in the following answers have been rounded to six decimal places.

Position Function: $s(t) = -4.967840t^2 - 0.237435t + 0.291470$

Initial Height: $s_0 = 0.291470$ meter

Initial Velocity: $v_0 = -0.237435$ meter per second

Velocity Function: $v(t) = s'(t) = -9.935679t - 0.237435$

Acceleration Function: $a(t) = s''(t) = -9.935679$

3. No. The graph of the function passes through or touches only two of the data points. Evaluating the velocity function from Exercise 2 for the times given in the data table shows that the velocity function's error is about 0.1 meter per second or more at each time. This is too large of an error for a set of velocity data where the initial velocity is -0.16405 meter per second.

4. The values in the following answers have been rounded to six decimal places.

Velocity Function: $v(t) = -9.785271t - 0.146953$

Acceleration Function: $a(t) = v'(t) = -9.785271$

5. The velocity function from Exercise 4 is a better fit to the data. The graph of the Exercise 4 velocity function passes through more of the data points than the graph of the Exercise 2 velocity function. Evaluating the velocity function from Exercise 4 for the times given in the data table shows that the velocity function's error is less than 0.04 meter per second for a majority of the time. Of the two velocity functions, the velocity function from Exercise 4 has a more acceptable error.

6. The acceleration function from Exercise 4 is closer to the actual value of earth's gravity. The percent error is 0.002%. (The percent error for the acceleration function from Exercise 2 is 0.014%.)

7. The values in the following answers have been rounded to six decimal places.

Position Function: $s(t) = -4.756079t^2 + 2.589810t + 0.808799$

Initial Height: $s_0 = 0.808799$ meter

Initial Velocity: $v_0 = 2.589810$ meters per second

The ball was thrown. One reason is that the initial velocity is positive, implying that the initial movement of the ball is upward, which is not what a dropped ball would do. Also, analyzing the height data would reveal that the ball starts at about 0.8 meter, rises to a height of about 1.16 meters, and then falls to a height of about 0.6 meter. Again, this is not what a dropped ball would do and is characteristic of a thrown ball.

8. s' is positive for $t = 0$ to $t \approx 0.272$ and negative for $t \approx 0.272$ to $t = 0.62$. The graph of s' tells you that on the interval that s' is positive, the graph of the position function s increases, and on the interval that s' is negative, the graph of the position function s decreases.

9. The slope of the tangent line is horizontal when $t \approx 0.272$. The velocity of the ball when $t \approx 0.272$ is 0 meter per second.

LAB 4 BOYLE'S LAW

Differentiation

1.

Pressure, P (atm)	Volume, V (L)	PV
0.25	2.801	0.70025
0.50	1.400	0.7
0.75	0.9333	0.699975
1.00	0.6998	0.6998
2.00	0.3495	0.699
3.00	0.2328	0.6984
4.00	0.1744	0.6976
5.00	0.1394	0.697

The product of the pressure and the volume is approximately equal to 0.70. Looking at the graph, as the pressure increases the volume decreases and vice versa. The graph has asymptotes at $y = 0$ and $x = 0$. The pressure is never 0.

2. The data is nearly linear. The slope of the line that approximates the data is $m \approx 0.70$, which is the constant of proportionality k found in Exercise 1. The y-intercept occurs at the origin.

3. $PV = k$

$$V = \frac{k}{P}$$

$$\frac{dV}{dP} = -\frac{k}{P^2}$$

4. From Exercise 1, $k \approx 0.70$. Therefore,

$$\frac{dV}{dP} = -\frac{0.70}{P^2}.$$

The graph of the derivative is always negative because the graph of $PV = k$ is always positive. The slope of the derivative's tangent line is always positive while the graph of $PV = k$ is always positive.

5. $V = \dfrac{0.70}{P}$ was used to compute the unknown values in the following table. Student answers may vary on their estimate of k. The values in the table are accurate to four decimal places.

Pressure, P (atm)	Volume, V (L)	Pressure, P (atm)	Volume, V (L)
1.00	0.6998	3.00	0.2328
1.50	0.4667	3.50	0.2000
2.00	0.3495	4.00	0.1744
2.50	0.2800	4.50	0.1556

6. From Boyle's Law, we know $P_1V_1 = k$ and $P_2V_2 = k$. Therefore, $P_1V_1 = P_2V_2$. Solving for V_2 yields

$$V_2 = \frac{P_1V_1}{P_2}.$$

7. The values in the table are accurate to four decimal places.

Pressure, P (atm)	Volume, V (L)	Pressure, P (atm)	Volume, V (L)
1.00	0.6998	3.00	0.2328
1.50	0.4665	3.50	0.1995
2.00	0.3495	4.00	0.1744
2.50	0.2796	4.50	0.1550

8. From Charles's Law, we know the initial volume $V_1 = mT_1$ and the new volume $V_2 = mT_2$. Solving both equations for m yields

$$m = \frac{V_1}{T_1} \quad \text{and} \quad m = \frac{V_2}{T_2}.$$

Therefore, $\dfrac{V_2}{T_2} = \dfrac{V_1}{T_1}$. Solving for V_2 yields $V_2 = \dfrac{V_1T_2}{T_1}$.

9. From combining Boyle's Law and Charles's Law, we know the initial volume

$$V_1 = k\frac{T_1}{P_1} \quad \text{and the new volume} \quad V_2 = k\frac{T_2}{P_2}.$$

Solving both equations for k yields $k = \dfrac{V_1P_1}{T_1}$ and $k = \dfrac{V_2P_2}{T_2}$. Therefore, $\dfrac{V_2P_2}{T_2} = \dfrac{V_1P_1}{T_1}$. Solving for V_2 yields .

$$V_2 = \frac{V_1P_1T_2}{T_1P_2}.$$

LAB 5 PACKAGING

Optimization

1.

Product	Surface Area (in.^{2})
Baking powder	38.4845
Cleanser	81.5400
Coffee	87.6033
Coffee creamer	78.6969

Product	Surface Area (in.^{2})
Frosting	53.5635
Pineapple juice	116.1133
Soup	41.6575
Tomato puree	77.8015

2. Yes, it is possible to design another baking powder container with a surface area of 38.48 square inches and a volume of 17.92 cubic inches. Let r and h be the radius and height, respectively, of another container with a surface area of 38.48 square inches and a volume of 17.92 cubic inches. Then

$$38.48 = 2\pi r^2 + 2\pi rh \text{ and } 17.92 = \pi r^2 h.$$

Solving for h in the volume formula yields

$$h = \frac{17.92}{\pi r^2}.$$

Substituting into the surface area formula yields

$$38.48 = 2\pi r^2 + 2\pi r\left(\frac{17.92}{\pi r^2}\right).$$

Solving for r yields two positive solutions and one negative solution (which can be disregarded). The positive solutions are $r \approx 1.25$ inches and $r \approx 1.60$ inches. The height of the container with a radius of 1.60 inches is approximately 2.23 inches.

3. The surface area of a cylinder is $S = 2\pi r^2 + 2\pi rh$ and the volume is $V = \pi r^2 h$. Solving for h in the volume formula yields

$$h = \frac{V}{\pi r^2}.$$

Substituting for h in the surface area formula yields

$$S = 2\pi r^2 + 2\pi r\left(\frac{V}{\pi r^2}\right),$$

which simplifies to

$$S = 2\pi r^2 + \frac{2V}{r}.$$

If you substitute the volume of one of the containers for V and graph the resulting equation, you can see the container could have a smaller surface area because the minimum of the graph does not occur at the container's radius.

4. Using the equation relating surface area and volume,

$$S = 2\pi r^2 + \frac{2V}{4},$$

let $V = 17.92$ to get

$$S = 2\pi r^2 + \frac{2(17.92)}{r} = 2\pi r^2 + \frac{35.84}{r}.$$

The derivative of S with respect to r is

$$\frac{dS}{dr} = 4\pi r - \frac{35.84}{r^2}.$$

Setting the derivative equal to zero and solving for r yields the following.

$$4\pi r - \frac{35.84}{r^2} = 0$$

$$4\pi r^3 - 35.84 = 0$$

$$r^3 = \frac{35.84}{4\pi}$$

$$r \approx 1.4181$$

The radius of the baking powder container that minimizes surface area is $r \approx 1.4181$ inches, which has a surface area of 37.9093 inches. This radius is larger than the radius given in the lab's Data. The radius given in the lab's Data does not minimize surface area because $r = 1.25$ is not a solution of the minimization problem above.

5. The answers for Radius, Height, and Surface Area in the table below have been rounded to four decimal places. For each volume V, the radius r was found by minimizing

$$S = 2\pi r^2 + \frac{2V}{r}.$$

The height h was found using the formula $h = \frac{V}{\pi r^2}$. The surface area S was found using the formula $S = 2\pi r^2 + 2\pi rh$.

Product	**Volume (in.^{3})**	**Radius (in.)**	**Height (in.)**	**Surface Area (in.^{2})**
Cleanser	49.54	1.9903	3.9808	74.6711
Coffee62.12	2.1463	4.2924	86.8297	
Coffee creamer	48.42	1.9752	3.9505	73.5412
Frosting	30.05	1.6848	3.3696	53.5054
Pineapple juice	92.82	2.4537	4.9074	113.4865
Soup	20.18	1.4754	2.9509	41.0327
Tomato puree	52.56	2.0300	4.0597	77.6733

6. The answers for Side of Base and Surface Area in the table below have been rounded to four decimal places. For each volume V and height h, the side of the base x was found using the formula

$$x = \sqrt{\frac{V}{h^2}}.$$

The surface area S was found using the formula $S = x^2 + 4xh$.

Volume (in.^{3})	Height (in.)	Side of Base (in.)	Surface Area (in.^{2})
17.92	3.65	2.2158	37.2605
49.54	7.50	2.5701	83.7086
62.12	5.20	3.4563	83.8372
48.42	6.85	2.6587	79.9171
30.05	3.60	2.8892	49.9519
92.82	6.70	3.7221	113.6063
20.18	3.80	2.3045	40.3391
52.56	4.40	3.4562	72.7744

The rectangular containers for coffee, frosting, pineapple juice, soup, and tomato puree have a smaller surface area than the corresponding cylindrical containers. The rectangular containers for baking powder, cleanser, and coffee creamer have a larger surface area than the corresponding cylindrical containers. Students may list several advantages for either type of container. Check the reasonableness of these advantages.

7. To minimize surface area for a fixed volume V, you could solve for h in $V = x^2h$ and substitute for h in the surface area equation $S = x^2 + 4xh$ to obtain

$$S = x^2 + 4x\left(\frac{V}{x^2}\right),$$

then use the revised surface area equation to find the minimum. To minimize volume for a fixed surface area S, you could solve for h in $S = x^2 + 4xh$ and substitute for h in the volume equation to obtain

$$V = x^2\left(\frac{S - x^2}{4x}\right),$$

then use the revised volume equation to find the minimum.

8. Answers will vary. Some reasons might be that the cylinder with the optimal surface area isn't as pleasing to the eye, doesn't display the product label very well, or, in the case of soda cans, fit the human hand as well as the cylinder that doesn't use the optimal surface area.

LAB 6 WANKEL ROTARY ENGINE

Area

1. Approximately 11 square inches. This estimate was obtained by counting the number of squares inside the rotor. There are approximately 176 squares that lie entirely or partially within the rotor. Since each square has an area of $\frac{1}{4} \cdot \frac{1}{4} = \frac{1}{16}$ in.^{2}, the area of the rotor is $176 \cdot \frac{1}{16} = 11$ in.^{2} Other reasonable methods of obtaining an estimate, such as the one used in Exercise 2, are certainly acceptable.

2. The inscribed triangle has sides of length 4 inches and its area is

$$\frac{\sqrt{3} \cdot (4)^2}{4} = 4\sqrt{3} \approx 6.93 \text{ in.}^2$$

The circumscribed triangle has sides of 6 inches and its area is

$$\frac{\sqrt{3} \cdot (6)^2}{6} = 6\sqrt{3} \approx 10.39 \text{ in.}^2$$

The estimate from Exercise 1 should lie between these two areas.

3. The approximate value of the integral is 1.45. The area of the triangle is

$$\frac{\sqrt{3} \cdot (4)^2}{4} = 4\sqrt{3} \approx 6.93 \text{ in.}^2$$

Therefore, the area of the rotor is

$$(3 \cdot \text{Area of Shaded Region}) + \text{Area of Equilateral Triangle} = (3 \cdot 1.45) + 6.93 = 11.28 \text{ in.}^2$$

The result is reasonable because it is greater than Exercise 2's underestimate and less than Exercise 2's overestimate.

4. The estimate of the area is approximately 1.432 in.^{2}. The error in the estimate is the same as the error E in the Trapezoidal Rule, which is given by

$$E \le \frac{(b-a)^3}{12n^2}[\max|f''(x)|], \quad a \le x \le b.$$

The maximum value of $|f''(x)|$ on the interval $[0, 4]$ is $|f''(2)| \approx 0.536$. For $n = 10$,

$$E \le \frac{(4-0)^3}{12(10)^2}(0.536) \approx 0.0286.$$

The error is less than 3%. The acceptability of the error depends on the manufacturing standards. To reduce the error, the region being measured should be divided into more sections.

The error in the estimate of the region's area can be reduced by increasing the number of measurements taken to increase the number of intervals used for the Trapezoidal Rule.

5. The estimate of the area is approximately 1.445 in.^{2}. Simpson's Rule will be better. As the number of intervals increases, the error in Simpson's Rule decreases more rapidly than the error in the Trapezoidal Rule.

6. Using Simpson's Rule, the estimate of the area is 0.816 in.^{2}. Using the Trapezoidal Rule, the estimate of the area is 0.810 in.^{2}. Either estimate can be used to approximate the area of the entire housing chamber by using the symmetry of the housing chamber as shown in the Data section. Thus, the estimate can be multiplied by four to obtain the approximation of the entire housing chamber.

LAB 7 NEWTON'S LAW OF COOLING

Exponential Decay

1. Initially, when the difference between the water's temperature and room temperature was great, the rate of cooling was large. This rate decreased over time, approaching zero as the water's temperature approached room temperature. As for the temperature of the water reaching 69.548°F, some students may conclude from the graph that the temperature will approach but never reach 69.548°F. Some students may say yes, concluding that the difference between the water's temperature and room temperature will eventually become small enough to be unmeasureable. Other reasonable explanations are acceptable.

2. The general solution is $y = L + Ce^{kt}$. In real-life terms, y could be the temperature of water (or cooling plastic), L could be the temperature of air, and t could be the time in seconds. The arbitrary constant C is the difference between the temperature of the object and the temperature of the surrounding medium. Note: If a student provides an alternate version of the general solution given above, make sure they are equivalent.

3. **Differential equation:** $\dfrac{dy}{dt} = k(y - 69.548)$

General solution: $y = 69.548 + Ce^{kt}$

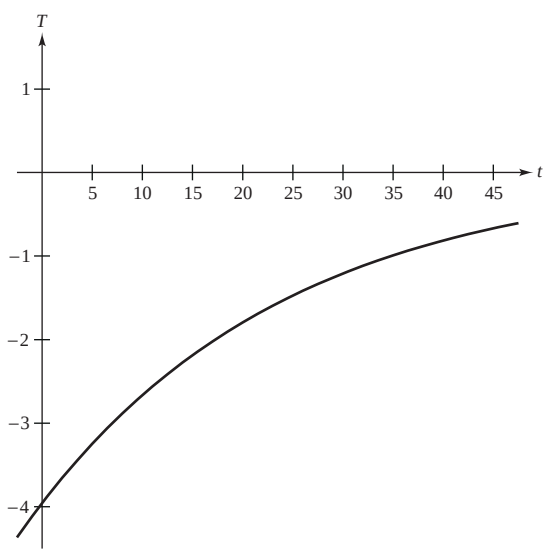
Particular solution: $y = 69.548 + 100.08e^{-0.0395t}$

The value of C should be calculated by solving $169.628 = 69.548 + Ce^{k(0)}$ for C. The value of k should be calculated by solving $87.134 = 69.548 + 100.08e^{k(44)}$ for k.

The arbitrary constant C is the difference between the temperature of the object and the temperature of the surrounding medium, which confirms the description given in the answer to Exercise 2.

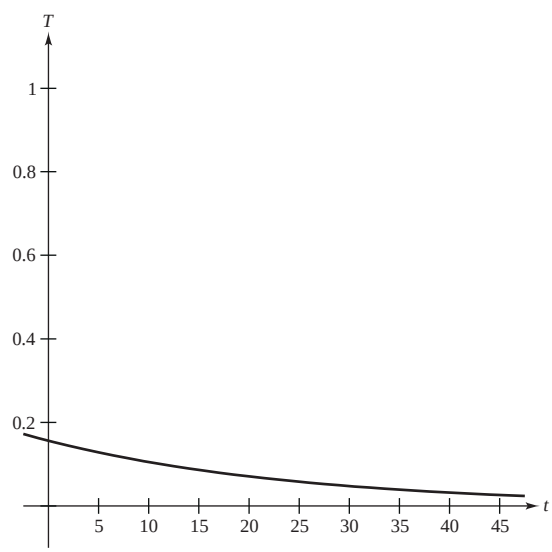
4. Since the term $e^{-0.0395t}$ in the particular solution approaches but never reaches zero, the temperature of the water doesn't reach room temperature. However, over time, the difference between the water's temperature and room temperature will eventually become small enough to be unmeasureable.

5. The graph of the first derivative is negative, approaching zero as t approaches infinity.



The value of the first derivative at time t is the slope of the tangent line.

6. The graph of the second derivative is positive, approaching zero as t approaches infinity.



The positive value of the second derivative at time t means the graph of the function y is concave upward.

7. *Answer for Maple, Mathematica, and Derive lab manuals:*

The equations found using *Maple, Mathematica,* or *Derive* should be similar to the ones given in the answer to Exercise 3, except they may have a higher level of accuracy. If the equations are different from the student's equations, one reason might be that they wrote the general solution differently (which would have made the particular solution different). Even though the equations may look different, they should be equivalent.

Answer for Mathcad and Graphing Utility lab manuals:

The numerical solution found using *Mathcad* or a graphing utility is usually within two-three degrees of the actually temperature. The worst approximation is about five degrees off. If the solution produces different temperature approximations than the student's particular solution, a possible reason is round-off error.

8. Differential equation: $\dfrac{dy}{dt} = k(y - 58)$

General solution: $y = 58 + Ce^{kt}$

Particular solution: $y = 58 + 242e^{kt}$

There isn't enough information to determine the value of the proportionality constant k because an appropriate temperature at a time other than $t = 0$ needs to be given.

9. One possible solution is to keep the chiller system at a cooler temperature (less than 58°F). The viability of this solution depends on the effect the cooler temperature has on the plastic. Also, there would be a higher cost of keeping the chiller system at a cooler temperature.

LAB 8 STRETCHING A SPRING

Hooke's Law

1. The graph of $F = kx$ is a straight line. k is the slope of the line and is the change in the force F divided by the change in the distance x.

2. Since k is the slope, you can use two of the data points to calculate k. Thus,

$$k = \frac{\Delta F}{\Delta x} = \frac{22.73 - 0.00}{1.37 - 0.00} \approx 16.59.$$

This value of k provides a model that is a good fit to the data.

3. The value of k would be valid only for springs of the same size and strength.

4. The spring would be damaged (or broken) and lose its ability to return to its original shape. The relationship between F and x would no longer be linear.

5. Using the value of k from Exercise 2,

$$W = \int_a^b 16.59x\,dx.$$

The graph of W will look like part of a parabola, because the relationship between work done to stretch the spring and the distance x the spring is stretched is quadratic. The relationship is quadratic because F is linear and thus its integral will be quadratic.

6. The values of work in the following table were calculated using $W = \int_a^b 16.59x\,dx$ and letting $a = 0$ and b a distance value in the table.

Distance (in meters)	Work (in newton-meters)
3	74.655
6	298.620
9	671.895
12	1194.480
15	1866.375

Check the graph provided by students. It should accurately reflect the data in the table and be quadratic.

7. $W = \int_0^{3.22} 16.59x\,dx = \Big[8.295x^2\Big]_0^{3.22} \approx 86.0059$ newton-meters.

The work done in stretching the spring from $x = 3.22$ meters to $x = 6.44$ meters is more than 86.0059 newton-meters because of the quadratic relationship between work and distance.

8. *Answer for Maple, Mathematica, and Derive lab manuals:*

The solution of the differential equation is $x(t) = 5\cos(3.1305t) - 2.2361\sin(3.1305t)$. The spring bounces up and down.

Answer for Mathcad and Graphing Utility lab manuals: The spring bounces up and down.

LAB 9 CONSTRUCTING AN ARCH DAM

Surface Area, Volume, and Fluid Force

1. The cross sectional area can be approximated by using geometric formulas, namely the area of a triangle and the area of a trapezoid. The cross section of the dam on $-70 \le x \le -16$ is nearly triangular, so the area can be estimated using the area of a triangle formula. The base is $-16 - (-70) = 54$ feet and the height is 244 feet. Thus, the area is

$$\tfrac{1}{2} \cdot 54 \cdot 244 = 6588 \text{ square feet.}$$

The cross section of the dam on $-16 \le x \le 59$ is a trapezoid with bases of 16 feet and $59 - (-16) = 75$ feet and a height of 389 feet. Thus, the area is

$$\tfrac{389}{2}(16 + 75) = 17{,}699.5 \text{ square feet.}$$

The total area is $6588 + 17{,}699.5 = 24{,}287.5$ square feet. This total is not exact because of the approximation of the cross section of the dam on $-70 \le x \le -16$. Note: Students may come up with alternate methods.

2. To calculate the area of a cross section, integrate the piecewise function f on each interval and add the results.

$$\begin{aligned}\int_{-70}^{59} f(x)\,dx &= \int_{-70}^{-16}(0.03x^2 + 7.1x + 350)\,dx + \int_{-16}^{0} 389\,dx + \int_0^{59}(-6.593x + 389)\,dx \\ &= 23{,}502.7235\end{aligned}$$

The area is 23,502.7235 square feet. If the model of the cross section is exact, then the area is exact. If the model is an approximation of the cross section, then the area is an approximation.

3. If the values are not exactly the same, they may be slightly different because of round-off error. If there is a great difference between the values, an error may have been made in answering Exercise 2.

4. The volume of the arch dam can be calculated using the shell method as follows.

$$\begin{aligned}\frac{150}{360}2\pi\int_{-70}^{59}(x + 220)f(x)\,dx &= \frac{5}{12}2\pi\left(\int_{-70}^{-16}(x + 220)(0.03x^2 + 7.1x + 350)\,dx + \int_{-16}^{0}(x + 220)(389)\,dx\right. \\ &\qquad \left. + \int_0^{59}(x + 220)(-6.593x + 389)\,dx\right) \\ &\approx 1.35 \times 10^7\end{aligned}$$

The volume is approximately 1.35×10^7 ft^3.

5. If the values are not exactly the same, they may be slightly different because of round-off error. If there is a great difference between the values, an error may have been made in answering Exercise 4.

6. The major axis is $140 - 112 = 28$ feet long, so $2a = 28 \implies a = 14$. The minor axis is 16 feet long, so $2b = 16 \implies b = 8$. The equation of the ellipse is

$$\frac{x^2}{8^2} + \frac{y^2}{14^2} = 1, \text{ or } \frac{x^2}{64} + \frac{y^2}{196} = 1.$$

Solving for x yields $x = 8\sqrt{1 - \frac{y^2}{196}} = \frac{4}{7}\sqrt{196 - y^2}$. Because the horizontal length of the gate is $2x$,

$$L(y) = \frac{8}{7}\sqrt{196 - y^2}.$$

7. The center of the gate is at $389 - (140 + 112)/2 = 263$ feet. Thus, the depth $h(y)$ of the water at y feet is $h(y) = 263 - y$.

8. The integral for fluid force on the gate is as follows.

$$\begin{aligned} F &= w\int_c^d h(y)L(y)\,dy \\ &= 62.4\int_{-14}^{14}(263 - y)\left(\frac{8}{7}\sqrt{196 - y^2}\right)dy \\ &\approx 5{,}774{,}000 \end{aligned}$$

The fluid force on the gate (rounded to the nearest thousand) is approximately 5,774,000 pounds. This approximation of the integral can be obtained using *Derive*, *Maple*, *Mathcad*, *Mathematica*, or a graphing utility.

9. The pressure will be the same and can be found as follows. The equation of the ellipse is

$$\frac{x^2}{14^2} + \frac{y^2}{8^2} = 1, \text{ or } \frac{x^2}{196} + \frac{y^2}{64} = 1.$$

Solving for x yields

$$x = 14\sqrt{1 - \frac{y^2}{64}} = \frac{7}{4}\sqrt{64 - y^2}.$$

Because the horizontal length of the gate is $2x$,

$$L(y) = \frac{7}{2}\sqrt{64 - y^2}.$$

The center of the gate is at $389 - (134 + 118)/2 = 263$ feet. Thus, the depth $h(y)$ of the water at y feet is $h(y) = 263 - y$. The integral for fluid force on the gate is given below.

$$\begin{aligned} F &= w\int_c^d h(y)L(y)\,dy \\ &= 62.4\int_{-8}^{8}(263 - y)\left(\frac{7}{2}\sqrt{64 - y^2}\right)dy \\ &\approx 5{,}774{,}000 \end{aligned}$$

The fluid force on the gate (rounded to the nearest thousand) is approximately 5,774,000 pounds. This approximation of the integral can be obtained using *Derive*, *Maple*, *Mathcad*, *Mathematica*, or a graphing utility.

LAB 10 CALCULUS OF THE MERCATOR MAP

Integration

1. The expression

$$\sum_{i=1}^{n} R\Delta\phi \sec \phi_i$$

approximates how far from the equator to draw the line representing the latitude ϕ_n. Thus, the distance of a latitude line from the equator is given by

$$\int_0^a R \sec \phi \, d\phi.$$

For example, on a globe with a 6 inch radius, the latitude line at 15° (which is $\pi/12$ radians) is

$$\int_0^{\pi/12} 6 \sec \phi \, d\phi \approx 1.589 \text{ inches.}$$

Latitude Line (in degrees north of the equator)	Distance from Equator
15°	1.589
30°	3.296
45°	5.288
60°	7.902
75°	12.166
90°	Infinitely large

2. For the latitude line 90° north of the equator, the integral does not converge. This latitude represents the north pole.

3. Note: Proofs may vary. Students should validate each step.

$$\begin{aligned}\int \sec \phi \, d\phi &= \int \sec \phi \left(\frac{\sec \phi + \tan \phi}{\sec \phi + \tan \phi}\right) d\phi \\ &= \int \frac{\sec^2 \phi + \sec \phi \tan \phi}{\sec \phi + \tan \phi} \, d\phi\end{aligned}$$

Let $u = \sec \phi + \tan \phi$ and $du = \sec^2 \phi + \sec \phi \tan \phi \, d\phi$ and substitute as follows.

$$\begin{aligned}\int \sec \phi \, d\phi &= \int \frac{\sec^2 \phi + \sec \phi \tan \phi}{\sec \phi + \tan \phi} \, d\phi \\ &= \int \frac{1}{u} \, du \\ &= \ln|u| + C \\ &= \ln|\sec \phi + \tan \phi| + C\end{aligned}$$

4. The results are not identical. However, the results can be shown to be equivalent through numerical evaluation, graphing both results on the same set of axes, and using trigonometric identities and laws of logarithms.

5. (a)
$$\int \sec\phi\, d\phi = \int \frac{1}{\cos\phi}\, d\phi = \int \frac{1}{\sin\left(\frac{\pi}{2} - \phi\right)}\, d\phi$$

Let
$$u = \frac{1}{2}\left(\frac{\pi}{2} - \phi\right) \text{ and } du = -\frac{1}{2}\, d\phi$$
and substitute as follows.
$$\begin{aligned}\int \sec\phi\, d\phi &= \int \frac{1}{\sin\left(\frac{\pi}{2} - \phi\right)}\, d\phi \\ &= \int \frac{-2}{\sin(2u)}\, du \\ &= -\int \frac{1}{\sin u \cos u}\, du \\ &= -\int \frac{1}{\frac{\sin u}{\cos u}\cos^2 u}\, du \\ &= -\int \frac{\sec^2 u}{\tan u}\, du\end{aligned}$$

Let $v = \tan u$ and $dv = \sec^2 u\, du$ and substitute as follows.
$$\begin{aligned}\int \sec\phi\, d\phi &= -\int \frac{\sec^2 u}{\tan u}\, du \\ &= -\int \frac{1}{v}\, dv \\ &= -\ln|v| + C \\ &= -\ln|\tan u| + C \\ &= -\ln\left|\tan\left[\frac{1}{2}\left(\frac{\pi}{2} - \phi\right)\right]\right| + C\end{aligned}$$

(b)
$$\int \sec\phi\, d\phi = \int \frac{1}{\cos\phi}\, d\phi = \int \frac{1}{\sin\left(\frac{\pi}{2} + \phi\right)}\, d\phi$$

Let
$$u = \frac{1}{2}\left(\frac{\pi}{2} + \phi\right) \text{ and } du = \frac{1}{2}\, d\phi$$
and substitute as follows.
$$\begin{aligned}\int \sec\phi\, d\phi &= \int \frac{1}{\sin\left(\frac{\pi}{2} + \phi\right)}\, d\phi \\ &= \int \frac{2}{\sin(2u)}\, du \\ &= \int \frac{1}{\sin u \cos u}\, du \\ &= \int \frac{1}{\frac{\sin u}{\cos u}\cos^2 u}\, du \\ &= \int \frac{\sec^2 u}{\tan u}\, du\end{aligned}$$

Let $v = \tan u$ and $dv = \sec^2 u\, du$ and substitute as follows.
$$\begin{aligned}\int \sec\phi\, d\phi &= \int \frac{\sec^2 u}{\tan u}\, du \\ &= \int \frac{1}{v}\, dv \\ &= \ln|v| + C \\ &= \ln|\tan u| + C \\ &= \ln\left|\tan\left[\frac{1}{2}\left(\frac{\pi}{2} + \phi\right)\right]\right| + C\end{aligned}$$

6. The integral could be written as $\frac{\pi}{180^\circ}\int \sec\left(\phi \cdot \frac{\pi}{180^\circ}\right) d\phi$.

7. No. An inch of map distance at 30° north represents a miles and an inch of map distance at 60° north represents b miles. Thus,
$$\frac{a}{b} = \frac{\sec 30^\circ}{\sec 60^\circ} \text{ and } a \neq b.$$

8. A Mercator map should not be used to compare the areas of two regions. The further a region is from the equator, the more a Mercator map "stretches" the earth-distance. Thus, regions far from the equator on a Mercator map will appear to be much larger than they actually are. The distortion of the earth-distances would also make determining the distance between two cities difficult.

LAB 11 KOCH SNOWFLAKE

Fractals

1.

Stage	Sides	Perimeter	Area
0	3	3	$\frac{\sqrt{3}}{4}$
1	12	4	$\frac{\sqrt{3}}{3}$
2	48	$\frac{16}{3}$	$\frac{10\sqrt{3}}{27}$
3	192	$\frac{64}{9}$	$\frac{94\sqrt{3}}{243}$
4	768	$\frac{256}{27}$	$\frac{862\sqrt{3}}{2187}$
5	3072	$\frac{1024}{81}$	$\frac{7822\sqrt{3}}{19{,}683}$
$\vdots$			
n	$3(4)^n$	$3\left(\frac{4}{3}\right)^n$	$\frac{\sqrt{3}}{4}\left[1+\frac{1}{3}\sum_{i=0}^{n-1}\left(\frac{4}{9}\right)^i\right]$

2. Yes. The Koch snowflake is an example of a closed region in the plane that has a finite area and an infinite perimeter. As n approaches infinity, the area after the nth iteration is

$$\frac{\sqrt{3}}{4}\left[1+\frac{1}{3}\sum_{i=0}^{n-1}\left(\frac{4}{9}\right)^i\right],$$

which is a geometric series and converges to

$$\frac{2\sqrt{3}}{5}.$$

The perimeter after the nth iteration is $3\left(\frac{4}{3}\right)^n$, which approaches infinity as n approaches infinity.

3.

Stage	Sides	Perimeter	Area
0	4	4	1
1	16	$\frac{16}{3}$	$1+\frac{\sqrt{3}}{9}$
2	64	$\frac{64}{9}$	$1+\frac{13\sqrt{3}}{81}$
3	256	$\frac{256}{27}$	$1+\frac{133\sqrt{3}}{729}$
4	1024	$\frac{1024}{81}$	$1+\frac{1261\sqrt{3}}{6561}$
5	4096	$\frac{4096}{243}$	$1+\frac{11{,}605\sqrt{3}}{59{,}049}$
$\vdots$			
n	4^{n+1}	$4\left(\frac{4}{3}\right)^n$	$1+\frac{\sqrt{3}}{9}\sum_{i=0}^{n-1}\left(\frac{4}{9}\right)^i$

4. The nth iteration will have $5(4)^n$ iterations. The Koch snowflake that starts with a pentagon has a greater area than the other two versions. The initial polygon has a greater area, and each iteration adds more triangles (and thus more area) than the other two versions.

5. After the nth iteration, the perimeter for the Koch snowflake that starts with a pentagon is $5\left(\frac{4}{3}\right)^n$. After the nth iteration, the perimeter in Exercise 1 was $3\left(\frac{4}{3}\right)^n$ and in Exercise 3 was $4\left(\frac{4}{3}\right)^n$. Thus, the Koch snowflake that starts Koch snowflake that starts with a pentagon has a greater perimeter than the other two versions at any iteration n.

6. More than one viewing window can be found to reproduce the graphs in the lab. One possible viewing window for each graph is given below.

Derive

Upper left: $x\text{min} = 0.042$, $x\text{max} = 0.13$, $y\text{min} = 0.072$, $y\text{max} = 0.17$
Upper right: $x\text{min} = -0.25$, $x\text{max} = -0.325$, $y\text{min} = -0.002$, $y\text{max} = 0.024$
Lower left: $x\text{min} = 0.25$, $x\text{max} = 0.3$, $y\text{min} = -0.002$, $y\text{max} = 0.072$
Lower right: $x\text{min} = -0.15$, $x\text{max} = -0.10$, $y\text{min} = -0.002$, $y\text{max} = 0.024$

Maple

Upper left: $x\text{min} = 18$, $x\text{max} = 50$, $y\text{min} = 30$, $y\text{max} = 70$
Upper right: $x\text{min} = -131$, $x\text{max} = -111.4$, $y\text{min} = -1$, $y\text{max} = 10$
Lower left: $x\text{min} = 100$, $x\text{max} = 117$, $y\text{min} = 1$, $y\text{max} = 30$
Lower right: $x\text{min} = -61$, $x\text{max} = -44$, $y\text{min} = -1$, $y\text{max} = 9.6$

Mathcad

Upper left: $x\text{min} = 0.167$, $x\text{max} = 0.5$, $y\text{min} = 0.289$, $y\text{max} = 0.674$
Upper right: $x\text{min} = -1.31$, $x\text{max} = -1.11$, $y\text{min} = -0.02$, $y\text{max} = 0.096$
Lower left: $x\text{min} = 1$, $x\text{max} = 1.17$, $y\text{min} = 0$, $y\text{max} = 0.29$
Lower right: $x\text{min} = -0.61$, $x\text{max} = -0.44$, $y\text{min} = -0.01$, $y\text{max} = 0.096$

Mathematica

Upper left: $x\text{min} = 0.167$, $x\text{max} = 0.5$, $y\text{min} = 0.289$, $y\text{max} = 0.674$
Upper right: $x\text{min} = -1.31$, $x\text{max} = -1.11$, $y\text{min} = -0.02$, $y\text{max} = 0.096$
Lower left: $x\text{min} = 1$, $x\text{max} = 1.17$, $y\text{min} = 0$, $y\text{max} = 0.29$
Lower right: $x\text{min} = -0.61$, $x\text{max} = -0.44$, $y\text{min} = -0.01$, $y\text{max} = 0.096$

Graphing Utility

Upper left: $x\text{min} = 0.167$, $x\text{max} = 0.5$, $y\text{min} = 0.289$, $y\text{max} = 0.674$
Upper right: $x\text{min} = -1.31$, $x\text{max} = -1.11$, $y\text{min} = -0.02$, $y\text{max} = 0.096$
Lower left: $x\text{min} = 1$, $x\text{max} = 1.17$, $y\text{min} = 0$, $y\text{max} = 0.29$
Lower right: $x\text{min} = -0.61$, $x\text{max} = -0.44$, $y\text{min} = -0.01$, $y\text{max} = 0.096$

LAB 12 ANALYZING A BOUNCING TENNIS BALL

Infinite Series

1. An exponential decay model fits the bouncing ball data because the rate of change of the height is initially very high but decreases rapidly after a few bounces and approaches zero as the number of bounces increases. A model that fits the data is $y = 100e^{[(\ln 0.0045)/9]x}$. A student's equation may be rounded, but they should closely approximate this equation. Check that a student's graph properly represents their model and fits the data well.

2. The value used for y_0 is 100, the initial height. The value used for p is 0.55. The value of p can be found by dividing the height after the first bounce by the initial height of the ball. Answers will vary. For the answers given here, this model and the model given in the answer to Exercise 1 both model the data reasonably well. One advantage of the model in this exercise is the additional information about the bounce of the tennis ball given by the model, the rebound height, as compared to the model in Exercise 1, which gives only the initial height.

3. Because of factors such as friction and air resistance, the tennis ball will eventually come to a stop. As the number of bounces n increases, the exponential model approaches zero and eventually evaluates to a value that would be unmeasurable.

4. Add the initial height and the sum of two times each other height.

$$100 + 2 \cdot 55 + 2 \cdot 30.25 + 2 \cdot 16.64 + 2 \cdot 9.15 + 2 \cdot 5.03 + 2 \cdot 2.77 + 2 \cdot 1.52$$
$$+ 2 \cdot 0.84 + 2 \cdot 0.46 \approx 343.32 \text{ inches}$$

5. This method is more exact than the method given in Exercise 4. The approximate value of D is $344.\overline{4}$. This answer is slightly more than the answer in Exercise 4 because D takes sums more bounces of the tennis ball than the sum in Exercise 4.

6. The total vertical distance includes both the distance the ball travels on its way down and the distance the ball travels on its way up.

7. No, this can not be a USTA sanctioned tennis ball. Using the result from the equation $y = 100(0.58)^n$, where 0.58 is the maximum rebound rate allowed by the USTA, the maximum vertical distance a sanctioned USTA tennis ball could travel after five bounces would be

$$D = 100 + 2\sum_{n=1}^{5} 100(0.58)^n \approx 358 \text{ inches.}$$

8. The minimum rebound rate is 0.53 inches, the initial height is 100 inches, and the equation is $y = 100(0.53)^n$. Thus, the minimum vertical distance a USTA sanctioned tennis ball could travel is

$$D = 100 + 2\sum_{n=1}^{\infty} 100(0.53)^n \approx 325.5 \text{ inches.}$$

The maximum rebound rate is 0.58 inches, the initial height is 100 inches, and the equation is $y = 100(0.58)^n$. Thus, the maximum vertical distance a USTA sanctioned tennis ball could travel is

$$D = 100 + 2\sum_{n=1}^{\infty} 100(0.58)^n \approx 376.2 \text{ inches.}$$

9. The total vertical distance D traveled by the tennis ball is approximately 688.9 inches when the height is 200 inches. Thus, when the height doubled, the vertical distance traveled doubled. If the initial height is multiplied by a positive integer k, then D becomes

$$D = 100k + 2\sum_{n=1}^{\infty} (100k)(0.55)^n = k\left(100 + 2\sum_{n=1}^{\infty} 100(0.55)^n\right).$$

Thus, when the initial height is multiplied by a positive integer k, the total vertical distance D traveled by the tennis ball increases by a factor of k.

LAB 13 COMETS

Conics

1. A polar equation to model Comet Hale-Bopp's orbit is

$$r = \frac{ed}{1 + e\sin\theta}.$$

The eccentricity e is given in the Data section as approximately 0.995. The perihelion distance 0.914 AU occurs when $\theta = \pi/2$. Thus,

$$0.914 = \frac{(0.995)d}{1 + (0.995)\sin(\pi/2)} \implies d \approx 1.8326.$$

The polar equation for Comet Hale-Bopp's orbit is

$$r = \frac{1.823437}{1 + 0.995\sin\theta}.$$

Because the perihelion distance occurs when $\theta = \pi/2$, the value of θ that corresponds to the aphelion distance is $\theta = 3\pi/2$. The aphelion distance is

$$r = \frac{1.823437}{1 + 0.995\sin(3\pi/2)} \approx 364.687 \text{ AU.}$$

2. With θ varying from $\pi/2$ to $3\pi/2$, the graph displays half of the orbit. With θ varying from $\pi/2$ to $5\pi/2$, the graph displays the entire orbit. With θ varying from $\pi/2$ to $9\pi/2$, the graph displays the entire orbit traced twice.

3. The eccentricity e of the ellipse, half the length of the major axis a, and the distance c from the center of the ellipse to its focus are related by $e = c/a$. Because the perihelion distance is 0.914 AU, $a - c = 0.914$. Thus,

$$0.995 = \frac{a - 0.914}{a} \implies a \approx 182.8 \quad \text{and} \quad c \approx 181.886.$$

To calculate half the length of the minor axis b, use $c^2 = a^2 - b^2 \implies b = \sqrt{a^2 - c^2}$. Thus,

$$b = \sqrt{182.8^2 - 181.886^2} \approx 18.257.$$

The center (h, k) of the ellipse is $(0, -181.886)$. The parametric equations are

$$x = 18.257 \cos\theta \quad \text{and} \quad y = 182.8 \sin\theta - 181.886.$$

The graphs of the parametric equations with θ varying as described in Exercise 2 should be the same as the graphs of the polar equation.

4. Using the values of a and b found in Exercise 3, a rectangular equation for Comet Hale-Bopp's orbit is

$$\frac{x^2}{18.257^2} + \frac{y^2}{182.8^2} = 1.$$

The choice of which way to represent Comet Hale-Bopp's orbit is left to personal preference. Check the logic of the reasoning given for the choice.

5. The value of $\sqrt{2GM/p}$ for Comet Hale-Bopp is

$$\sqrt{\frac{2(6.67 \times 10^{-11})(1.991 \times 10^{30})}{(0.914)(1.496 \times 10^{11})}} \approx 44{,}073.$$

If Comet Hale-Bopp was traveling at about 44 km/sec or less when it reached perihelion, then the orbit was an ellipse because $v = 44$ km/sec $= 44{,}000$ m/sec, which is less than 44,073. For Comet Hale-Bopp's orbit to be parabolic for the same value of p, the velocity at perihelion would have to be

$$\sqrt{\frac{2(6.67 \times 10^{-11})(1.991 \times 10^{30})}{(0.914)(1.496 \times 10^{11})}} \approx 44{,}073 \text{ meters per second.}$$

For Comet Hale-Bopp's orbit to be hyperbolic for the same value of p, the velocity at perihelion would have to be greater than

$$\sqrt{\frac{2(6.67 \times 10^{-11})(1.991 \times 10^{30})}{(0.914)(1.496 \times 10^{11})}} \approx 44{,}073 \text{ meters per second.}$$

6. Answers will vary. Two reasons are the gravitational pull of the planets and collisions with other objects.

7. As Comet Hale-Bopp approaches perihelion distance, the comet's velocity approaches its maximum. As Comet Hale-Bopp approaches aphelion distance, the comet's velocity approaches its minimum.

8. $t \approx 1.2$ years or approximately 438 days.

LAB 14 SUSPENSION BRIDGES

Parabolas

1. Place the origin of the coordinate axes at the lowest point of the cable midway between the two towers so that $(h, k) = (0, 0)$. Thus, the cable passes through the point $(2130, 390)$ and $2130^2 = 4p(390) \implies p \approx 2908$. An equation that describes the hanging cable is $x^2 = 4(2908)y \implies x^2 = 11{,}632y$.`

2. The value of w is 10,800 pounds per foot, the cable passes through the point $(2130, 390)$ and

$$390 = \frac{10{,}800(2130)^2}{2\|\mathbf{T}_0\|} \implies \|\mathbf{T}_0\| \approx 62{,}800{,}000 \quad \text{(rounded to the nearest hundred thousand).}$$

Thus, an equation that describes the hanging cable is

$$y = \frac{10{,}800x^2}{2(62{,}800{,}000)} \implies y = \frac{27x^2}{314{,}000}.$$

3. The equations are approximately the same. If the values of p in Exercise 1 and $\|\mathbf{T}_0\|$ in Exercise 2 are not rounded, the equations would be the same. The equations are two different ways to represent the same parabola.

4. The result of the integral is $s \approx 4353$ feet. The length of the cable is reasonable because it is longer than the span of the bridge but not so much longer that it would need a greater sag. The length of the cable from the lowest point to either support is about $4353/2 = 2176.5$.

5. In each case, let the origin occur at the lowest point between the two supports. When the sag height is 386 feet, the cable passes through the point $(2130, 386)$ and

$$386 = \frac{10{,}800(2130)^2}{2\|\mathbf{T}_0\|} \implies \|\mathbf{T}_0\| \approx 63{,}500{,}000 \text{ (rounded to the nearest hundred thousand).}$$

Thus, an equation that describes the hanging cable when the sag height is 386 feet is

$$y = \frac{10{,}800x^2}{2(63{,}500{,}000)} \implies y = \frac{27x^2}{317{,}500}.$$

The length of the cable when the sag height is 386 feet is approximately 4351 feet. When the sag height is 394 feet, the cable passes through the point $(2130, 394)$ and

$$394 = \frac{10{,}800(2130)^2}{2\|\mathbf{T}_0\|} \implies \|\mathbf{T}_0\| \approx 62{,}200{,}000 \text{ (rounded to the nearest hundred thousand).}$$

Thus, an equation that describes the hanging cable when the sag is 394 feet is

$$y = \frac{10{,}800x^2}{2(62{,}200{,}000)} \implies y = \frac{27x^2}{311{,}000}.$$

The length of the cable when the sag height is 394 feet is approximately 4355 feet. The sag height of 386 feet occurs in winter (the cold temperatures cause the cable to contract) and the sag height of 394 feet occurs in summer (the hot temperatures cause the cable to expand).

6. Using the values from Exercise 2, $\|\mathbf{T}\| = \sqrt{62{,}800{,}000 + 10{,}800x^2}$. The maximum tension occurs at both main supports, while the minimum tension occurs at the midway point between the main supports. For a sag height of 386 feet, $\|\mathbf{T}\| = \sqrt{63{,}500{,}000 + 10{,}800x^2}$ and the minimum tension at the midway point between the supports is approximately 7969 pounds. The maximum tension, which occurs at the main supports, is approximately 9301 pounds. For a sag height of 394 feet, $\|\mathbf{T}\| = \sqrt{62{,}200{,}000 + 10{,}800x^2}$ and the minimum tension at the midway point between the supports is approximately 7887 pounds. The maximum tension, which occurs at the main supports, is approximately 9231 pounds.

7. *For a sag height of 386 feet*

The minimum occurs at $x = 0$ and the tangent line is $y = 0$. The maximums occur at $x = 2130$ and $x = -2130$ and the tangent lines are

$$y = \frac{5751}{15{,}875}(x - 1065) \text{ and } y = -\frac{5751}{15{,}875}(x + 1065).$$

For a sag height of 390 feet

The minimum occurs at $x = 0$ and the tangent line is $y = 0$. The maximums occur at $x = 2130$ and $x = -2130$ and the tangent lines are

$$y = \frac{5751}{15{,}700}(x - 1065) \text{ and } y = -\frac{5751}{15{,}700}(x + 1065).$$

For a sag height of 394 feet

The minimum occurs at $x = 0$ and the tangent line is $y = 0$. The maximums occur at $x = 2130$ and $x = -2130$ and the tangent lines are

$$y = \frac{5751}{15{,}550}(x - 1065) \text{ and } y = -\frac{5751}{15{,}550}(x + 1065).$$

The absolute value of the tangent line's slope becomes greater as the tension increases. The direction of the tension is along the tangent line. Solving $\tan\theta = m$ for θ, where m is the slope of the tangent line.

8. Solving the given formula for θ should produce approximately the same results as Exercise 7. Round-off error may affect the results.

9. (a) Let the lowest point occur at the origin and the supports at $(x_0, 75)$ and $(x_1, 125)$ where $x_0 < 0$ and $x_1 > 0$. To determine the x-coordinates of the supports, substitute each point into

$$y = \frac{800x^2}{2\|\mathbf{T}_0\|},$$

solve the equations for $\|\mathbf{T}_0\|$ and set them equal to each other to obtain $\frac{16}{3}x_0^2 = \frac{16}{5}x_1^2$. Since $x_1 - x_0 = 1000$, substitute to obtain $\frac{16}{3}x_0^2 = \frac{16}{5}(x_0 + 1000)^2$. Simplifying yields $\frac{2}{3}x_0^2 - 2000x_0 - 1{,}000{,}000 = 0$. Solving for x_0 we obtain approximately -436. Thus, $x_1 = 564$. The cable passes through the point $(564, 125)$ and

$$125 = \frac{800(564)^2}{2\|\mathbf{T}_0\|} \Longrightarrow \|\mathbf{T}_0\| \approx 1{,}017{,}907.$$

Thus, an equation that describes the hanging cable is

$$y = \frac{800x^2}{2(1{,}017{,}907)} \Longrightarrow y = \frac{400x^2}{1{,}017{,}907}.$$

(b) The lowest point is at $(0, 0)$ and the highest point is at $(564, 125)$.

(c) The total length of the cable is

$$\int_{-436}^{564} \sqrt{1 + \left(\frac{800x}{1{,}017{,}907}\right)^2}\, dx \approx 1026 \text{ feet.}$$

The length of the cable from the lowest point to the highest point is

$$\int_{0}^{564} \sqrt{1 + \left(\frac{800x}{1{,}017{,}907}\right)^2}\, dx \approx 582 \text{ feet.}$$

LAB 15 RACE-CAR CORNERING

Vector-Valued Functions

1. $\mathbf{v}(t) = -212.66\sin(0.4759t)\mathbf{i} + 212.66\cos(0.4759t)\mathbf{j}$

$\mathbf{a}(t) = -101.20\cos(0.4759t)\mathbf{i} - 101.20\sin(0.4759t)\mathbf{j}$

The speed of the race car as it moves through the turn is

$$\|\mathbf{v}(t)\| \approx \sqrt{(-212.66\sin(0.4759t))^2 + (212.66\cos(0.4759t))^2} = 212.66.$$

The speed of the car is constant, but the direction keeps changing. Thus the velocity, which comprises both speed and direction, is not constant. The acceleration of the race car as it moves through the turn is

$$\|\mathbf{a}(t)\| \approx \sqrt{(-101.20\cos(0.4759t))^2 + (-101.20\sin(0.4759t))^2} = 101.20.$$

Because **a** has both magnitude and direction and the direction of the acceleration keeps changing, it is not constant.

2. $\mathbf{v}(t) = -220.01\sin(0.4211t)\mathbf{i} + 220.01\cos(0.4211t)\mathbf{j}$

$\mathbf{a}(t) = -92.65\cos(0.4211t)\mathbf{i} - 92.65\sin(0.4211t)\mathbf{j}$

The speed of the race car as it moves through the turn is

$$\|\mathbf{v}(t)\| \approx \sqrt{(-220.01\sin(0.4211t))^2 + (-220.01\cos(0.4211t))^2} = 220.01.$$

The speed of the car is constant, but the direction keeps changing. Thus the velocity, which comprises both speed and direction, is not constant. The acceleration of the race car as it moves through the turn is

$$\|\mathbf{a}(t)\| \approx \sqrt{(-92.65\cos(0.4211t))^2 + (-92.65\sin(0.4211t))^2} = 92.65.$$

Because **a** has both magnitude and direction and the direction of the acceleration keeps changing, it is not constant.

3. $\mathbf{v}(t) = -278.66\sin(0.3797t)\mathbf{i} + 278.66\cos(0.3797t)\mathbf{j}$

$\mathbf{a}(t) = -105.81\cos(0.3797t)\mathbf{i} - 105.81\sin(0.3797t)\mathbf{j}$

The speed of the race car as it moves through the turn is

$$\|\mathbf{v}(t)\| = \sqrt{(-278.66\sin(0.3797t))^2 + (-278.66\cos(0.3797t))^2} = 278.66.$$

The speed of the car is constant, but the direction keeps changing. Thus the velocity, which comprises both speed and direction, is not constant. The acceleration of the race car as it moves through the turn is

$$\|\mathbf{a}(t)\| = \sqrt{(-105.81\cos(0.3797t))^2 + (-105.81\sin(0.3797t))^2} = 105.81.$$

Because **a** has both magnitude and direction and the direction of the acceleration keeps changing, it is not constant.

4. Each curvature has been rounded to four decimal places.

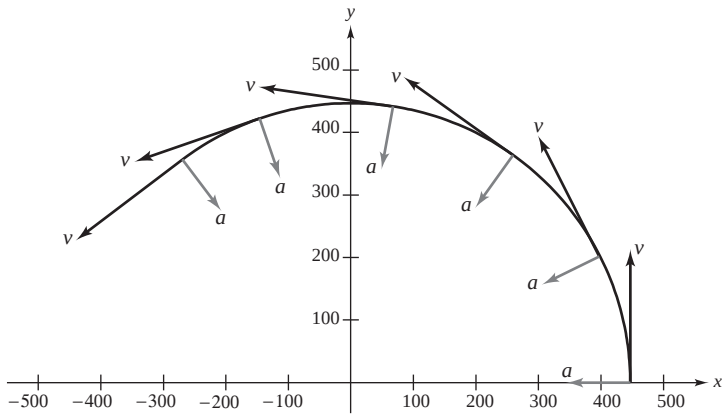
Curvature of Turn 1: 0.0022
Curvature of Turn 2: 0.0019
Curvature of Turn 3: 0.0014

Turn 3 is the easiest since it has the smallest curvature and the magnitude of the race car's velocity vector relative to the other turns is greatest in Turn 3. Turn 1 is the hardest since it has the largest curvature and the magnitude of the race car's velocity vector relative to the other turns is the smallest in Turn 1.

5. The answers in the following table have been rounded to two decimal places.

t	0.00	1.00	2.00	3.00	4.00	4.66
$x(t)$	446.85	397.20	259.27	63.72	−145.98	−269.32
$y(t)$	0.00	204.72	363.94	442.28	422.33	356.57

6.



The velocity vectors all lie on the tangent line at the given point. The acceleration vectors all point to the origin, which is the center of the circle that the turn is an arc of.

7. The following answers have been rounded to the nearest integer.

Turn 1: Approximately 182,172

Turn 2: Approximately 166,762

Turn 3: Approximately 182,170

LAB 16 PUTTING A SHOT

Projectile Motion

1. The length of time the shot will remain in the air can be found by setting the vertical component of the position function

$$\mathbf{r}(t) = (v_0 \cos\theta)t\mathbf{i} + \left[h + (v_0 \sin\theta)t - \frac{1}{2}gt^2\right]\mathbf{j}$$

equal to zero feet as follows.

$$h + (v_0 \sin\theta)t - \frac{1}{2}gt^2 = 0 \implies \frac{1}{2}gt^2 - (v_0 \sin\theta)t - h = 0$$

Using the Quadratic Formula to solve for t yields the following. (**Note:** the negative solution can be discarded.)

$$t = \frac{-(-v_0 \sin\theta) + \sqrt{(-v_0 \sin\theta)^2 - 4((1/2)g)(-h)}}{2((1/2)g)} \implies t = \frac{v_0 \sin\theta + \sqrt{v_0{}^2 \sin^2\theta + 2gh}}{g}$$

2. Set x equal to the horizontal component of the position function

$$\mathbf{r}(t) = (v_0 \cos\theta)t\mathbf{i} + \left[h + (v_0 \sin\theta)t - \frac{1}{2}gt^2\right]\mathbf{j}$$

and substitute the length of time t the shot put is in the air to determine the horizontal distance traveled.

$$\begin{aligned} x &= (v_0 \cos\theta)\left(\frac{v_0 \sin\theta + \sqrt{v_0{}^2 \sin^2\theta + 2gh}}{g}\right) \\ &= \frac{v_0 \cos\theta}{g}\left(v_0 \sin\theta + \sqrt{v_0{}^2\left(\sin^2\theta + \frac{2gh}{v_0{}^2}\right)}\right) \\ &= \frac{v_0 \cos\theta}{g}\left(\sin\theta + \sqrt{\sin^2\theta + \frac{2gh}{v_0{}^2}}\right) \end{aligned}$$

3. (a) Increasing the initial height h slightly increases the horizontal distance traveled by the shot put.

(b) Increasing the initial speed v_0 increases the horizontal distance traveled by the shot put.

(c) To maximize horizontal distance, a shot putter should focus on maximizing the initial speed since maximizing the initial height has a small effect on the distance traveled. Also, the initial height is nearly fixed by the height of the shot putter.

4. For integer values of θ, angles of 41°–43° produce the maximum horizontal distance (for the given initial conditions) of 19.77 meters.

5. $25 = (2.1)\tan 2\theta_{max} \implies \theta_{max} \approx 0.74$ radians or 42.6°. This value is within the range given in Exercise 4 and is more accurate.

6. The angles in the following table are rounded to two decimal places. Students may or may not provide answers with greater (or less) accuracy.

Year	Distance (in meters)	Angle (in degrees)
1983	22.22	42.30
1985	22.62	42.35
1986	22.64	42.35
1987	22.79	42.37
1987	22.81	42.37
1987	22.91	42.38
1988	23.06	42.40
1990	23.12	42.41

The angles in the table fall with the range of angles found in Exercise 4, and are slightly less than the angle found in Exercise 5. Note that the best throw happens to be the closest to the angle in Exercise 5.

7. The distances in the following table have been rounded to two decimal places.

Year	Distance (in meters)
2002	24.55
2012	25.80
2022	27.12
2032	28.51
2042	29.97

Opinions of the model's accuracy will vary. Check the logic of student's reasoning.

8. The angles in the following table have been rounded to two decimal places.

Year	Distance (in meters)
2002	42.56
2012	42.67
2022	42.79
2032	42.89
2042	43.00

The angles in this table are greater than those in Exercise 6.

LAB 17 SATELLITE DISHES, FLASHLIGHTS, AND SOLAR ENERGY COLLECTORS

Using the Reflective Property of Parabolas

1. The receiver should be placed at the focus of the paraboloid so all incoming signals will be reflected to the receiver.

2. The satellite dish has a radius of 10 feet and a depth of 3.5 feet, so it must pass through the point $(10, 0, 3.5)$. Thus,

$$z = \frac{x^2}{a^2} + \frac{y^2}{a^2} \Longrightarrow 3.5 = \frac{10^2}{a^2} + \frac{0^2}{a^2} \Longrightarrow a = \sqrt{\frac{100}{3.5}}.$$

The equation for the satellite dish is

$$z = \frac{x^2}{\left(\sqrt{100/3.5}\right)^2} + \frac{y^2}{\left(\sqrt{100/3.5}\right)^2} = \frac{3.5x^2}{100} + \frac{3.5y^2}{100} = \frac{7x^2}{200} + \frac{7y^2}{200}.$$

The domain of the equation is the set of all points such that

$$0 \le \frac{7x^2}{200} + \frac{7y^2}{200} \le 3.5.$$

The range is $0 \le z \le 3.5$.

3. $$2\pi\int_0^r x\sqrt{1 + \left(\frac{x}{10}\right)^2}\,dx = \frac{\pi}{15}[(100 + r^2)^{3/2} - 1000]$$

The surface area is roughly proportional to the cube of the radius r.

4. For *Derive*, *Maple*, *Mathcad*, and *Mathematica*, the result of evaluating the surface area integral is different from the result in Exercise 3 by a constant. A graphing utility's result is the same as the result in Exercise 3. Student results will vary depending on the result obtained in Exercise 3. Generally, the results should be different only by a constant and should evaluate to the same result for a radius r, e.g. $r = 10$.

5. The focus is 1.5 centimeters from the vertex of the reflector, so $p = 1.5$ and $x^2 = 4(1.5)y = 6y$. A segment of the parabola that can be revolved about the y-axis to form the reflector is $0 \le x \le 4$. The radius is 4 centimeters and the surface area is

$$2\pi\int_0^4 x\sqrt{1 + \left(\frac{x}{3}\right)^2}\,dx = \frac{196}{9}\pi \text{ square centimeters.}$$

6. The graph in three dimensions is a parabolic cylinder with rulings parallel to the z-axis. The surface area can be found by multiplying the length of the trough times the length of the cross section.

$$18\int_{-10}^{10} \sqrt{1 + \left(\frac{2x}{25}\right)^2}\,dx \approx 114.61\pi \text{ square feet}$$

LAB 18 HYPERTHERMIA TREATMENTS FOR TUMORS

Volume

1. No. Since the tissue gets progressively hotter toward the center of the tumor, the center of the tumor will be hotter than the temperature of the equitherm at half of the radius of the tumor.

2. No, the value of V_T/V is

$$\frac{\frac{4}{3}\pi\left(\frac{1}{4}r\right)^3}{\frac{4}{3}\pi r^3} = \frac{\left(\frac{1}{4}r\right)^3}{r^3} = \frac{1}{64}.$$

If the radius of the tumor is 2.5 centimeters, the percent heated is 1.5625%.

3.

Radius of tumor that has reached an effective temperature	V_T	$\frac{V_T}{V}$
$\frac{1}{4}r$	$\frac{4}{3}\pi\left(\frac{1}{4}r\right)^3$	$\frac{1}{64}$
$\frac{1}{3}r$	$\frac{4}{3}\pi\left(\frac{1}{3}r\right)^3$	$\frac{1}{27}$
$\frac{1}{2}r$	$\frac{4}{3}\pi\left(\frac{1}{2}r\right)^3$	$\frac{1}{8}$

Radius of tumor that has reached an effective temperature	V_T	$\frac{V_T}{V}$
$\frac{2}{3}r$	$\frac{4}{3}\pi\left(\frac{2}{3}r\right)^3$	$\frac{8}{27}$
$\frac{3}{4}r$	$\frac{4}{3}\pi\left(\frac{3}{4}r\right)^3$	$\frac{27}{64}$
r	$\frac{4}{3}\pi r^3$	1

The value of V_T/V is the cube of the portion of r that has been heated.

(a) No. The portion of r that has been heated when $\frac{V_T}{V} = \frac{1}{2}$ is $\frac{1}{\sqrt[3]{2}}$.

(b) The portion of r that has been heated when $\frac{V_T}{V} = \frac{3}{4}$ is $\frac{\sqrt[3]{3}}{\sqrt[3]{4}}$.

4. The volume could be estimated by finding the volume of the spheres that inscribe and circumscribe the wrinkled sphere. The inscribed sphere has a radius of $r = 0.155$ and the circumscribed sphere has a radius of $r = 0.845$. Thus the radius of the wrinkled sphere is greater than

$$\frac{4}{3}\pi(0.155)^3 \approx 0.00497\pi \text{ cubic units}$$

and less than

$$\frac{4}{3}\pi(0.845)^3 \approx 0.80447\pi \text{ cubic units.}$$

Any estimate in this range would be valid.

5. $\displaystyle\int_0^{2\pi}\int_0^{\pi}\int_0^{0.5+0.345\sin 8\theta\sin\phi} \rho^2 \sin\phi\, d\rho\, d\phi\, d\theta$

$$= \frac{1}{3}\int_0^{2\pi}\int_0^{\pi}(0.5 + 0.345\sin 8\theta\sin\phi)^3 \sin\phi\, d\phi\, d\theta$$

$$= \frac{1}{3}\int_0^{2\pi}\int_0^{\pi}[0.125\sin\phi + 0.259\sin 8\theta\sin^2\phi + 0.179\sin^2 8\theta\sin^3\phi + 0.041\sin^3 8\theta\sin^4\phi]\, d\phi\, d\theta$$

$$= \frac{1}{3}\int_0^{2\pi}\Bigg[-0.125\cos\phi + 0.259\sin 8\theta\left(\frac{1}{2}\right)(\phi - \sin\phi\cos\phi) + 0.179\sin^2 8\theta\left(-\frac{\sin^2\phi\cos\phi}{3} - \frac{2}{3}\cos\phi\right)$$
$$+ 0.041\sin^3 8\theta\left(-\frac{\sin^3\phi\cos\phi}{4} + \frac{3}{8}(\phi - \sin\phi\cos\phi)\right)\Bigg]_0^{\pi} d\theta$$

$$\approx \frac{1}{3}\int_0^{2\pi}(0.25 + 0.1295\pi\sin 8\theta + 0.2387\sin^2 8\theta + 0.0154\pi\sin^3 8\theta)\, d\theta$$

$$= \frac{1}{3}\left[0.25\theta - \frac{0.1259\pi}{8}\cos 8\theta + 0.2387\left(\frac{\theta}{2} - \frac{\sin 8\theta\cos 8\theta}{16}\right) + 0.0154\pi\left(-\frac{\sin^2 8\theta\cos 8\theta}{24} - \frac{\cos 8\theta}{12}\right)\right]_0^{2\pi}$$

$$\approx 0.7736$$

6. The approximate value of the result is 0.7729. The difference between the answers to Exercises 5 and 6 can be attributed to round-off error.

7. $\displaystyle\int_0^{2\pi}\int_0^{\pi}\int_0^{0.75+0.35\sin 8\theta\sin 4\phi} \rho^2 \sin\phi\, d\rho\, d\phi\, d\theta$

$$= \frac{1}{3}\int_0^{2\pi}\int_0^{\pi}(0.75 + 0.35\sin 8\theta\sin 4\phi)^3 \sin\phi\, d\phi\, d\theta$$

$$\approx \frac{1}{3}\int_0^{2\pi}\int_0^{\pi}[0.422\sin\phi + 0.591\sin 8\theta\sin 4\phi\sin\phi + 0.276\sin^2 8\theta\sin^2 4\phi\sin\phi$$
$$+ 0.043\sin^3 8\theta\sin^3 4\phi\sin\phi]\, d\phi\, d\theta$$

$$= \frac{1}{3}\int_0^{2\pi}\Bigg[-0.422\cos\phi + 0.591\sin 8\theta\left(\frac{\sin 3\phi}{6} - \frac{\sin 5\phi}{10}\right) + 0.276\sin^2 8\theta\left(-\frac{\cos\phi}{2} - \frac{\cos 7\phi}{28} + \frac{\cos 9\phi}{36}\right)$$
$$+ 0.043\sin^3 8\theta\left(\frac{\sin 3\phi}{8} - \frac{3\sin 5\phi}{40} - \frac{\sin 11\phi}{88} + \frac{\sin 13\phi}{104}\right)\Bigg]_0^{\pi} d\theta$$

$$\approx \frac{1}{3}\int_0^{2\pi}(0.844 + 0.280\sin^2 8\theta)\, d\theta$$

$$= \frac{1}{3}\left[0.984 - 0.0175\cos 8\theta\sin 8\theta\right]_0^{2\pi}$$

$$\approx 2.0609$$

8. The approximate value of the result is 2.0604. The difference between the answers to Exercises 7 and 8 can be attributed to round-off error.

LAB 19 MATHEMATICAL SCULPTURES

Parametric Surfaces

Answers for *Maple, Mathcad, and Mathematica:*

1. The *x*- and *y*-equations have been interchanged. The torus appears to spiral in the opposite direction of the one shown in the lab's Data.

2. The *x*- and *z*-equations have been interchanged. The torus appears to spiral in the opposite direction of the one shown in the lab's Data and is centered around the *x*-axis instead of the *z*-axis.

3. The *y*- and *z*-equations have been interchanged. The torus appears to spiral in the opposite direction of the one shown in the lab's Data and is centered around the *y*-axis instead of the *z*-axis.

4. Yes, it is possible. Multiply either one of the *x*- and *y*-equations by a nonzero value, e.g. let the *x*-equation be

$$x = 3\sin u\left[7 + \cos\left(\frac{u}{3} - 2v\right) + 2\cos\left(\frac{u}{3} + \mathrm{v}\right)\right].$$

5. They both have only one edge, which can be traced more than once (twice for the Möbius strip and three times for *Umbilic Torus NC*) before returning to the starting point.

6. More than one orientation and more than one set of *u*- and *v*-ranges can be found to reproduce the graphs in the lab. One possible orientation and one set of *u*- and *v*-ranges for each graph is given below. The orientation is given in terms relevant to the program's 3D graphing language. The equations used for each graph are the same as the equations given in the lab's Data.

Maple

Upper left: orientation = [45, 90], $-\pi \le u \le \pi$, $-\pi \le v \le \pi$

Upper right: orientation = [90, 90], $-\pi \le u \le \pi$, $-\pi \le v \le \pi$

Lower left: orientation = [35, 235], $0 \le u \le \pi$, $-\pi \le v \le \pi$

Lower right: orientation = [160, 70], $0 \le u \le \frac{\pi}{2}$, $-\pi \le v \le \pi$

Mathcad

Upper left: rotation: 45, tilt: 0, $-\pi \le u \le \pi$, $-\pi \le v \le \pi$

Upper right: rotation: 90, tilt: 0, $-\pi \le u \le \pi$, $-\pi \le v \le \pi$

Lower left: rotation: 35, tilt: 235, $0 \le u \le \pi$, $-\pi \le v \le \pi$

Lower right: rotation: 160, tilt: 20, $0 \le u \le \frac{\pi}{2}$, $-\pi \le v \le \pi$

Mathematica

Upper left: ViewPoint –> {3.384, 0.000, 0.000}, $-\pi \le u \le \pi$, $-\pi \le v \le \pi$

Upper right: ViewPoint –> {0.000, 3.384, 0.000}, $-\pi \le u \le \pi$, $-\pi \le v \le \pi$

Lower left: ViewPoint –> {2.271, −1.590, 1.941}, $-\pi \le u \le 0$, $-\pi \le v \le \pi$

Lower right: ViewPoint –> {−2.988, 1.088, 1.157}, $0 \le u \le \frac{\pi}{2}$, $-\pi \le v \le \pi$

7. Values used to graph a hypocycloid with four arches will vary. One set of values is $a = 1$ and $b = 4$. Values used to graph a hypocycloid with five arches will vary. One set of values is $a = 1$ and $b = 5$. The relationship between a and b for a hypocycloid with three arches is $a = 3b$. The relationship between a and b for a hypocycloid with four arches is $a = 4b$. The relationship between a and b for a hypocycloid with n arches is $a = nb$.

Answers for *Derive* and *Graphing Utility*:

1. Yes, it is possible. Multiply either one of the x- and y-equations by a nonzero value, e.g. let the x-equation be

$$x = 3 \sin u\left[7 + \cos\left(\frac{u}{3} - 2v\right) + 2\cos\left(\frac{u}{3} + \mathrm{v}\right)\right].$$

2. They both have only one edge, which can be traced more than once (twice for the Möbius strip and three times for *Umbilic Torus NC*) before returning to the starting point.

3. The point of view for the upper graph is (10, 0, 0). The point of view for the lower graph is (0, 10, 0).

4. Values used to graph a hypocycloid with four arches will vary. One set of values is $a = 1$ and $b = 4$. Values used to graph a hypocycloid with five arches will vary. One set of values is $a = 1$ and $b = 5$. The relationship between a and b for a hypocycloid with three arches is $a = 3b$. The relationship between a and b for a hypocycloid with four arches is $a = 4b$. The relationship between a and b for a hypocycloid with five arches is $a = 5b$. The relationship between a and b for a hypocycloid with n arches is $a = nb$.

5.

a	*b*	*Number of Arches*
8	2	4
21	7	3
18	3	6
20	4	5

6. Solutions will vary. One solution is altering the u-range to be $-\pi \le u \le 0$ and viewing the torus from the point $(-2.3, 1.6, 1.9)$.

LAB 20 INTERACTING POPULATIONS

Euler's Method

1. The answers in the following table have been rounded to two decimal places.

k	*x*	*y*
0	55	10.00
1	80	10.34
2	105	11.85

The predator population is growing, but at a slower rate than the prey population.

2. Decreasing the magnitude of Δx improves the accuracy of Euler's method.

3. The estimate of y for $x = 80$ is $y \approx 10.89$ and for $x = 105$ is $y \approx 13.26$. It is an assumption that decreasing Δx improves the accuracy of the estimates. Assuming the accuracy is an improvement, the estimates are slightly higher than those in Exercise 1.

4. *Answer for Maple, Mathcad, Mathematica, and Derive lab manuals:*

Assuming that decreasing Δx improves the accuracy of the estimates, the graph of the solution from Exercise 3 is a better model of the interacting populations. The graph with the *Maple*, *Mathematica*, *Mathcad*, or *Derive* solution displays a model that has a higher estimate than the other two solutions as the time t increases.

Answer for Graphing Utility lab manual:

Assuming that decreasing Δx improves the accuracy of the estimates, the graph of the solution from Exercise 3 is a better model of the interacting populations.

5. Answers will vary. Some possible responses are given below.

- The prey has an abundant food supply.
  This is probably reasonable, but this assumption is not always true.
- The predator feeds exclusively on the prey.
  This assumption can be misleading. Most predators do not exclusively feed on one prey, but sometimes only one prey that the predator hunts is present.
- The environment can support unlimited quantities of the prey.
  This assumption does not seem reasonable.
- There is no need to consider females and males separately
  This assumption seems reasonable, unless some factor would kill one disproportionately.

6. Yes, the size of the predator population would be limited, especially if the predator feeds exclusively on the prey.

7. Yes, there is a point where the predator or prey population dies out. For example, if $y_0 = 0.1$ and $x_0 = 0.1$ for the situation described in Exercise 1, both y is approximately zero for $t = 11$. If the predator population dies out, the prey population will grow rapidly. Yes, there is a point where the prey population and the predator population die out. For example, if $y_0 = 10$ and $x_0 = 0.1$ for the situation described in Exercise 1, both y and x will be zero for $t = 19$.

8. Answers will vary depending on the values chosen.