LAB 16

PUTTING A SHOT

Projectile Motion

The men's shot put has been a test of strength for centuries. Early versions of the shot were heavy stones. Today's athletes use a shot made of metal weighing 7.26 kilograms (16 pounds). The men's shot put record by the end of 1960 was little more than 20 meters. Twenty eight years later, the record had moved beyond the 23-meter mark.

Observations

The angle of elevation of the shot's release is important to the athlete. If the angle is too steep, too much of the initial velocity will be used to send the shot up and horizontal distance will be sacrificed. If the angle is too low, the shot will not stay above ground long enough to travel very far.

Purpose

In this lab, you will analyze the men's shot put and the effect of different release heights and angles. You will use *Mathematica* to experimentally find an optimal angle of elevation to maximize the horizontal distance traveled by a shot. You will compare your experimental value with that of estimated angles of elevation used for world record shot puts.

References

For more information about the physics of a shot put, see "Dynamics: Throwing the Javelin, Putting the Shot" by M. S. Klamkin in the *The UMAP Journal* (Vol. VI, No. 2).

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Data

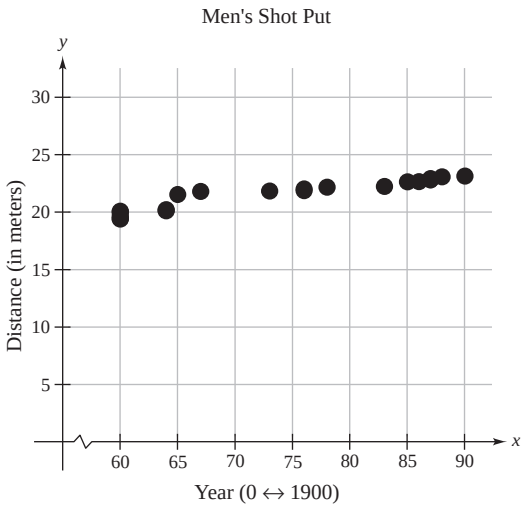
The year of a record set in the men's shot put and the record distance in meters is given in the table below. Let x represent the year, where $x = 0$ corresponds to 1900. Let y represent the record distance in meters.

Year, x	60	60	60	60	60	62	64	64
Distance, y	19.38	19.45	19.67	19.99	20.06	20.08	20.10	20.20

Year, x	64	65	67	73	76	76	78	83
Distance, y	20.68	21.52	21.78	21.82	21.85	22.00	22.15	22.22

Year, x	85	86	87	87	87	88	90
Distance, y	22.62	22.64	22.79	22.81	22.91	23.06	23.12

A scatter plot of the data is given below.



The data in the table and the scatter plot are stored in the *Mathematica* file called LAB16.NB.

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Exercises

Name _____

Date _____ Class _____

Instructor _____

1. ***Length of Time in the Air.*** Neglecting air resistance, the path of a shot put launched from an initial height h with initial speed v_0 and angle of elevation θ is described by the vector function

$$r(t) = (v_0 \cos \theta)t\mathbf{i} + \left[h + (v_0 \sin \theta)t - \frac{1}{2}gt^2\right]\mathbf{j}$$

where g is the gravitational constant. Show that the length of time t (in seconds) that the shot will remain in the air is given by the following.

$$t = \frac{v_0 \sin \theta + \sqrt{v_0^2 \sin^2 \theta + 2gh}}{g}$$

2. ***Horizontal Distance Traveled.*** Use the position function given in Exercise 1 to show that the horizontal distance x (in meters) traveled by the shot is given by the following.

$$x = \frac{v_0^2 \cos \theta}{g}\left(\sin \theta + \sqrt{\sin^2 \theta + \frac{2gh}{v_0^2}}\right)$$

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3. ***The Effect of Initial Height and Initial Speed.*** Use the horizontal distance x (in meters) given in Exercise 2 to answer the following.

a. What effect does increasing the initial height h have on the horizontal distance traveled?

b. What effect does increasing the initial speed v_0 have on the horizontal distance traveled?

c. To maximize horizontal distance, should a shot putter try to maximize the initial height, the initial speed, both or neither? Explain your reasoning.

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4. ***Finding the Optimal Angle Experimentally.*** What angle of elevation θ should the shot be released to maximize horizontal distance? Use *Mathematica* to experiment with different integer values of θ (in degrees) to produce a shot put with maximum horizontal distance if the release height is 2.1 meters and the initial velocity is 13.2 meters per second. What is the value of this angle?

5. ***Finding the Optimal Angle Analytically.*** The maximum horizontal distance x_{max} and the optimal angle of elevation θ_{max} are related by the following equation.

$$x_{max} = h \tan 2\theta_{max}$$

Use this equation to find the optimal angle for a shot released at a height of 2.1 meters that travels a horizontal distance of 25 meters. Does the value agree with the experimental value you determined in Exercise 4? Why or why not?

6. ***Angle of a World Record.*** Use the equation given in Exercise 5 to estimate the angle used for the world records given in the table below. (Assume the release height for each record is 2.1 meters.) Compare the angles. Do the angles nearly agree? How do these angles compare to the results of Exercises 4 and 5?

Year	Distance (in meters)	Angle (in degrees)
1983	22.22	
1985	22.62	
1986	22.64	
1987	22.79	
1987	22.81	
1987	22.91	
1988	23.06	
1990	23.12	

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7. ***Modeling the Data.*** An exponential equation was used to model the world records given in this lab's Data. Let x represent the year, where $x = 0$ corresponds to 1900. Let y represent the record distance in meters. The equation for the model is

$$y = 14.76(1.005)^x.$$

Use the model to estimate the world records for the following years. Do you think the model is accurate for all of the years? Why or why not?

Year	Distance (in meters)
2002	
2012	
2022	
2032	
2042	

8. ***World Record Angles.*** Use the equation given in Exercise 5 to estimate the angle used for the world records you estimated in Exercise 8. (Assume the release height for each record is 2.1 meters.) Compare the angles to those you found in Exercise 6.

Year	Angle (in degrees)
2002	
2012	
2022	
2032	
2042	

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